Ideas in Mathematics
Math 170, Spring 2016
Assignment 3, part 1

1. Indicate whether the following are always **true**, always **false**, or **sometimes** true and sometimes false, for any sets A, B, and C. The symbol $\subset$ should be understood to indicate a subset, not necessarily proper, as it was used in class.

- $\emptyset \subset A$ ________________
- $A \cap (B \cap C) = (A \cap B) \cap C$ ________________
- $A \subset \emptyset$ ________________
- $A \cup (B \cup C) = (A \cup B) \cup C$ ________________
- $A \subset (A \cup B)$ ________________
- $A \cap \emptyset = A$ ________________
- $A \subset (A \cap B)$ ________________
- $A \cup \emptyset = \emptyset$ ________________

2. Indicate whether the following are rational or irrational numbers. If they are rational, rewrite the given number in simplest form a/b, i.e., a and b are integers that have no common factors. A horizontal line above a group of digits indicates that that block of digits repeats ad infinitum.

- 100 ________________
- $10.23\overline{33}$ ________________
- $\sqrt{49}$ ________________
- $\sqrt{18}$ ________________
- 10.2 ________________
- $10.21\overline{21}$ ________________
- 10.213 ________________
- $5.214\overline{142}$ ________________

3. Indicate whether the following sets are **finite**, **countably infinite**, or **uncountable**. If they are countably infinite, you can indicate that by writing $\aleph_0$, the standard symbol for the cardinality $|\mathbb{N}|$. Let $\mathbb{P}$ indicate the set of primes.

- $\{\mathbb{N}, \mathbb{Q}, \mathbb{R}\}$ ________________
- $\{\mathbb{R} \cap \mathbb{N}\}$ ________________
- $\{\emptyset\}$ ________________
- $\{0 < x < 1 : x \in \mathbb{Q}\}$ ________________
- $\{a^n : a, n \in \mathbb{Z}\}$ ________________
- $\{1 < x < 2 : x \in \mathbb{R}\}$ ________________
- $\{(\mathbb{N} \cup \mathbb{Q}) \cup \mathbb{R}\}$ ________________
- $\{\mathbb{P} \cap \{2n : n \in \mathbb{Z}\}\}$ ________________

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Assignment 3, part 2

The following questions will require more thinking than those in part 1. Paper and pencil might help you try out certain ideas until you realize what works and what doesn't. As indicated in the syllabus, working with other people is highly encouraged. That said, the final answers you write down should be your own. Typed answers are preferred for part 2.

4. Prove that for any $n \in \mathbb{N}$, the number $n^3 - n$ is even.

5. Prove that $\sqrt{7}$ is irrational, i.e., it cannot be written as a/b for two whole numbers a and b.

6. Consider $\{\sqrt{x} : x \in \mathbb{Q}^+\} \cup \{\sqrt[3]{x} : x \in \mathbb{Q}^+\}$, the set of all square and cubed roots of positive rational numbers. Is this set countable? If yes, describe a 1-1 matching between this set and the set of natural numbers.

7. Can $\mathbb{Z}$ be divided into three sets A, B, and C so that each one is countably infinite and such that $A \cap B = \emptyset$, $B \cap C = \emptyset$, $C \cap A = \emptyset$, and $A \cup B \cup C = \mathbb{Z}$? Provide an example or explain why it is not possible.

8. In class we considered some infinite sets with the same cardinalities. For example, $|\mathbb{Z}| = |\mathbb{N}| = |\mathbb{Q}|$. The notion of cardinality, however, also applies to uncountable sets. Consider the two open intervals $(0, 1)$ and $(0, \infty)$, both of which are uncountable sets. Do these two sets have the same cardinality? If not, explain why they do not have the same cardinality; if yes, describe a one-to-one matching between elements of the two sets.