Ideas in Mathematics
Math 170, Spring 2016
Assignment 4

The following questions will require thinking. Paper and pen might help you try out ideas until you realize what works and what doesn't. As indicated in the syllabus, working with other people is highly encouraged. That said, the final answers you write down should be your own; typed answers are preferred.

In cases where you cannot think of a good answer, spend a few sentences explaining what you tried and why that didn't work out as you had hoped.

1. (a) Prove that every even number greater than or equal to 10 can be written as the sum of two unique composite numbers.

   (b) Prove that every odd number greater than or equal to 13 can be written as the sum of two unique composite numbers.

   These two proofs require some thinking, though they are not complicated; try doing several examples on scratch paper to see if you can determine a pattern.

2. With the exception of 2 and 3, consecutive numbers cannot both be prime, since at least one of them will be even. What about consecutive composite numbers? Think about whether we can find 5 consecutive composite numbers, or 5000 consecutive composite numbers. **Show that the numbers** $n!+2, n!+3, n!+4, \ldots, n!+n$ **are all composite.** This shows that we can find arbitrarily many consecutive numbers that are all composite.

   Remember that $n!$ indicates the factorial of a natural number n, which is the product of all natural numbers less than or equal to n. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

3. **BONUS:** (A in class, no deadline) Find a natural number $2^{(2^n)}+1$ that is prime for some $n \geq 5$.

4. **BONUS:** (5 points, no deadline) Prove that it is possible to draw a circle centered at $(\sqrt{2}, 1/3)$ that contains any number of points of the form (a, b) where a and b are both integers.