Ideas in Mathematics
Math 170, Spring 2016
Assignment 5, part 1

1. The notation $\sum_{i=1}^{n} x_i$ indicates a sum of terms $x_1, x_2, \ldots x_n$. Rewrite the following sums using $\sum$ notation.

   (a) $1 + 2 + \ldots + n$ $\sum_{i=1}^{n} i$

   (b) $1 - 2 + 3 - 4 \ldots n$

   (c) $x + x^2 + \ldots + x^n$

   (d) $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$

   (e) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots \frac{1}{n}$

   (f) $x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots + \frac{x^n}{n!}$

2. Evaluate each of the above sums for the given n.

   (a) Sum 1 for $n = 2$ 3

   (b) Sum 2 for $n = 3$

   (c) Sum 3 for $n = 4$

   (d) Sum 4 for $n = 5$

   (e) Sum 5 for $n = 6$

   (f) Sum 6 for $n = 7$

3. Draw the following graphs, given here by their vertices and edges.

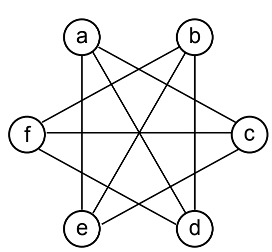
   (a) $V_1 = \{a, b, c\}$, $E_1 = \{\{a, b\}, \{c, b\}, \{a, c\}\}$

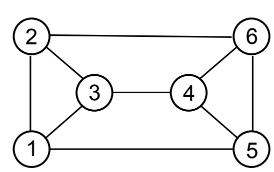
   (b) $V_2 = \{a, b, c, d\}$, $E_2 = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, a\}, \{a, c\}, \{b, d\}\}$

   (c) $V_3 = \{a, b, c, d, e, f\}$, $E_3 = \{\{a, b\}, \{c, d\}, \{e, f\}\}$

   (d) $V_4 = \{a, b, c, d, e, f\}$, $E_4 = \{\{a, d\}, \{a, e\}, \{a, f\}, \{b, d\}, \{b, e\}, \{b, f\}, \{c, d\}, \{c, e\}, \{c, f\}\}$

4. Describe each of the graphs below as a set of vertices and edges.





Ideas in Mathematics
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Assignment 5, part 2

The following questions will require more thinking than those in part 1. Paper and pencil might help you try out certain ideas until you realize what works and what doesn't. As indicated in the syllabus, working with other people is highly encouraged. That said, the final answers you write down should be your own. Typed answers are preferred for part 2.

5. In class we saw that the sum of degrees of all vertices is twice the number of edges, i.e.,

$$\sum_{i=1}^{|V|} deg(v_i) = 2|E|. \tag{1}$$

   Knowing that this sum is even, prove that in any graph, the number of vertices with odd degree must be even.

6. CORRECTED: Prove that a graph with $n = |V|$ vertices can have at most $(n^2 - n)/2$ edges.

7. A k**-regular graph** is a graph where each vertex has degree k. Draw a 2-regular graph on n vertices for every n in $\{3, 4, 5, 6, 7, 8\}$.

8. A graph is **connected** if every vertex is connected to every other vertex by some set of edges. Do regular graphs need to be connected? Either provide an example of a regular graph that is not connected, or else explain why this is not possible.

9. Look back to question 3. Notice that some of the graphs you drew had edges crossing one another, while others did not. Which of (a), (b), (c), and (d) can be drawn without edges crossing?

10. Read Chapter 6 "What Happens When Mathematics Gets into Position" from *The Language of Mathematics: Making the Invisible Visible*, by Keith Devlin.