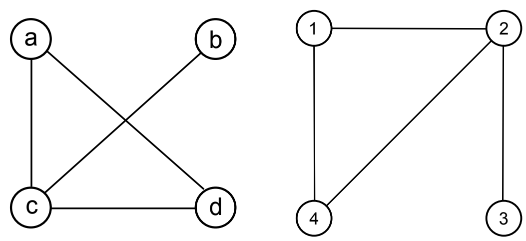
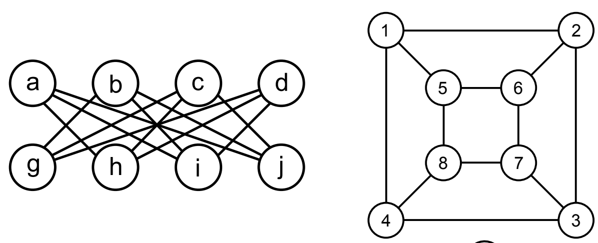
Ideas in Mathematics
Math 170, Spring 2016
Assignment 6, part 1

For each pair of graphs below, either indicate a 1-1 matching to show that the graphs are isomorphic (for example a-1, b-2, etc), or, if the graphs are *not* isomorphic, explain how you know they are not isomorphic.

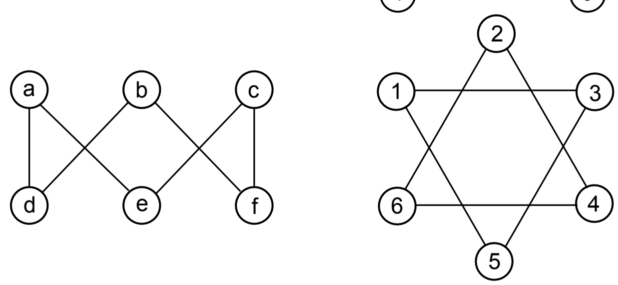
1.



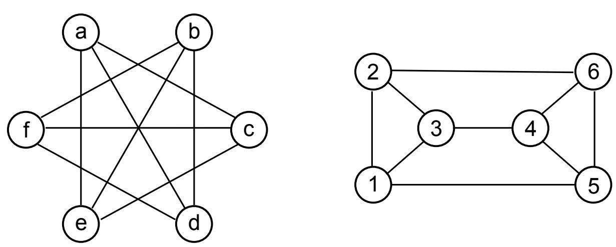
2.



3.



4.

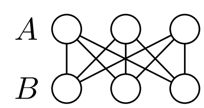


Ideas in Mathematics
Math 170, Spring 2016
Assignment 6, part 2

5. A **bipartite graph** is a graph whose vertices V can be divided into two sets A and B so that edges connect vertices in A to vertices in B, but do not connect vertices within A or within B. Two examples of bipartite graphs can be found in the posted course notes and in problem 7 below. If we know $|A|$ and $|B|$, what is the smallest possible value of E? What is largest possible value of $|E|$?

6. A bipartite graph can be planar. (a) Show that the graph associated with a cube is a bipartite graph; (b) explain why all faces in a bipartite planar graph must have degree at least 4.

7. A *complete bipartite* graph on vertex sets A and B is one in which every vertex in A is connected to every vertex in B. Below is illustrated the complete bipartite graph $K_{3,3}$, in two sets containing 3 vertices each.



Using Euler's theorem and your answer to 6, show that $K_{3,3}$ is non-planar.

8. In class we saw that K_5, the complete graph on 5 vertices, is non-planar. Suppose that we remove one edge from K_5. Is the resulting graph now planar? Either explain how you know that this graph is non-planar, or else draw this graph in such a way that no edges cross.

9. In class we saw that the sum of degrees of all faces is twice the number of edges, i.e.,

$$\sum_{i=1}^{|F|} deg(f_i) = 2|E|. \qquad (1)$$

Knowing that this sum is even, prove that there exists no planar graph with an odd number of faces with odd degree.

10. If you haven't done so yet, read Chapter 6 "What Happens When Mathematics Gets into Position" from *The Language of Mathematics: Making the Invisible Visible*, by Keith Devlin.