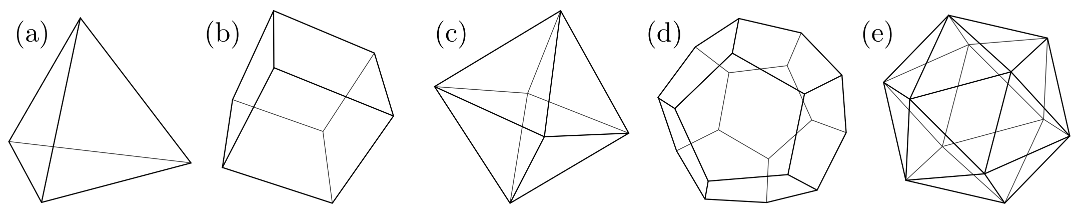
Ideas in Mathematics
Math 170, Spring 2016
Assignment 7, part 1

1. Below are illustrated each of the five Platonic solids. Name each and fill in the number of vertices v, edges e, and faces f, as well as the degree of each vertex $\deg(v_i)$, and the degree of each face $\deg(f_i)$.



Name of Polyhedron	v	e	f	$\deg(v_i)$	$\deg(f_i)$
(a)					
(b)					
(c)					
(d)					
(e)					

2. **Dual Polyhedra.** Choose a Platonic solid illustrated above and visualize the following. Place a vertex at the center of each face and connect pairs of these vertices if their corresponding faces share an edge. What polyhedron have you constructed? Repeat this exercise (on paper or in your mind) for each of the five Platonic solids and describe (on paper!) what you obtain for each.

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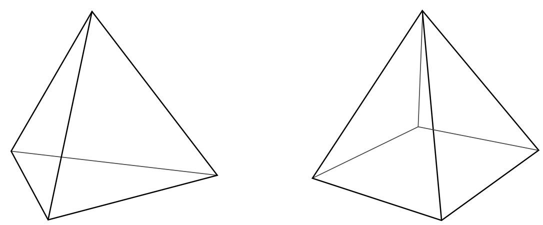
Assignment 7, part 2

3. A **simple** polyhedron is one such that exactly 3 faces meet at every corner. A tetrahedron and a cube are both simple, but a square pyramid is not, as four triangular faces meet at one of its corner. Prove that the graph of a simple polyhedron must have an even number of vertices.



4. Consider any graph G with n vertices. How many vertices in G can have degree 0? How many vertices in G can have degree n? Why can't G have two vertices u and v such that $\deg(u) = 0$ and $\deg(v) = n - 1$?

5. CORRECTED: Spend a few minutes trying to make a graph in which every vertex has a different degree. You will soon realize that this is not possible. Using results from question 4, explain why it is impossible for every vertex in a graph to have a distinct degree.

6. Use induction to prove

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}. \tag{1}$$