Ideas in Mathematics
Math 170, Spring 2016
Assignment 8, part 1

1. Pretend you're back in third grade. Solve the following division problems, writing out the quotient and remainder.

   - 7/3 2 r 1
   - 19/2
   - 75/19
   - 293/17
   - 183/23
   - 8911/239
   - $2^{15}/21$
   - $10^7/19$

2. Solve each congruence relation with a number in $\{0, 1, \ldots, m-1\}$, where m is the modulus.

   - $19 \equiv 6 \pmod{13}$
   - $-5 \equiv \quad \pmod{4}$
   - $1557 \equiv \quad \pmod{4}$
   - $-21 \equiv \quad \pmod{11}$
   - $23 \equiv \quad \pmod{13}$
   - $4^{13} \equiv \quad \pmod{17}$
   - $17^6 \equiv \quad \pmod{15}$
   - $17^{357} \equiv \quad \pmod{16}$

3. Complete the following multiplication table for arithmetic mod 7.

$\times$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

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4. Let n be an even integer. Show that $n^2 \equiv 0 \pmod 4$.

5. Let n be an odd integer. Show that $n^2 \equiv 1 \pmod 4$.

6. Using the results from Questions 4 and 5, prove that there are no integers a and b such that $a^2 + b^2 \equiv 3 \pmod 4$.

7. Prove that a number is divisible by 3 if and only if the sum of its digits (written in base 10) is divisible by 3. This proof is very similar to a proof we covered in class.

8. In arithmetic modulo m, we can define the square root of a number a to be a number b such that $a \equiv b^2 \pmod m$. For example, $1 \equiv 1^2 \pmod 5$ and $1 \equiv 4^2 \pmod 5$, so 1 and 4 are both "square roots" of 1 in arithmetic mod 5. For this problem, consider only square roots smaller than m.

   Consider arithmetic modulo m for $m = 6$, 9, and 12. For each m, list which numbers in $\{0, 1, \ldots, m-1\}$ have square roots, and list what those square roots are. Do some numbers in $\{0, 1, \ldots, m-1\}$ have more than 2 square roots? Do some numbers have only 1 square root? Do some numbers have no square roots?

9. Read Chapter 1 "Why Numbers Count" from *The Language of Mathematics: Making the Invisible Visible*, by Keith Devlin.