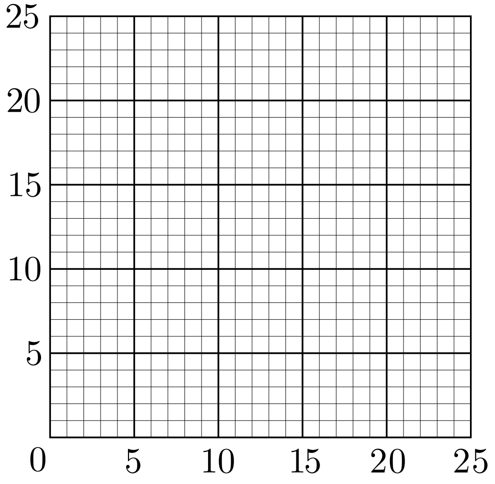
Ideas in Mathematics
Math 170, Spring 2016
Assignment 10, part 1

1. **Linear congruence generators** (LCG) generate pseudo-random numbers using modular arithmetic. For a fixed multiplier a, modulus m, and initial "seed" s, each "random" number is generated from the previous one using the recursive relation $x_{n+1} = ax_n \pmod{m}$; the first number is the seed, $x_0 = s$.

   For the given a, m, and s, compute x_0, x_1, x_2, x_3, x_4, and x_5.

   - $a = 3$, $m = 7$, $s = 4$ 4, 5, 1, 3, 2, 6
   - $a = 2$, $m = 13$, $s = 5$
   - $a = 10$, $m = 17$, $s = 4$
   - $a = 10$, $m = 15$, $s = 7$
   - $a = 11$, $m = 23$, $s = 3$
   - $a = 13$, $m = 137$, $s = 8$

2. **Spectral test.** Consider an LCG with $a = 11$ and $m = 23$. Choose any seed $0 < s < m$ and calculate all x_i. Then, on the graph below, plot all pairs of points (x_0, x_1), (x_1, x_2), $\ldots (x_{m-1}, x_0)$.



Ideas in Mathematics
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Assignment 10, part 2

3. Suppose every year, include leap years, has 365 days. Given that April 18, 2016 is a Monday, explain how you would use modular arithmetic to determine the day of the week for every subsequent year. In particular, compute the day of the week of April 18 in the years 2017, 2020, 2050, 2100, and 3000. Since the question ignores leap years, looking this up in an online calendar will give you the wrong answer.

4. The "random numbers" generated by Excel and most calculators are numbers between 0 and 1. How can you use an LCG, which naturally produces integers between 0 and $m - 1$, to generate numbers between 0 and 1?

5. Read Chapter 7 "How Mathematicians Figure the Odds" from *The Language of Mathematics: Making the Invisible Visible*, by Keith Devlin.