Ideas in Mathematics
Math 170, Spring 2016
Assignment 11, part 1

1. Indicate whether the given set is closed under the given binary operator.

   (a) $\mathbb{N}$, $a \times b$ closed
   (b) $\mathbb{Z}$, $a + b$
   (c) $\{0, 3, 6, 9\}$, $a + b \bmod 12$
   (d) $\{-2, -1, 0, 1, 2\}$, $a + b$
   (e) $\{1, 2, 4, 5, 7, 8\}$, $a \times b \bmod 9$
   (f) $\mathbb{Q}$, $(a + b)/2$

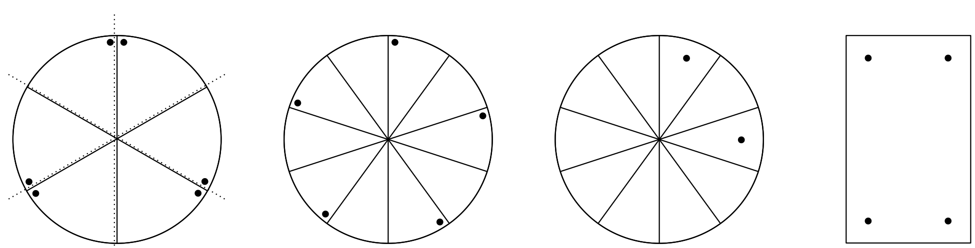
2. Indicate the identity element of the given set and operator, or state that there is no identity.

   (a) $\mathbb{N}$, $a \times b$ 1
   (b) $\mathbb{Z}$, $a + b$
   (c) $\{0, 3, 6, 9\}$, $a + b \bmod 12$
   (d) $\{-2, -1, 0, 1, 2\}$, $a + b$
   (e) $\{1, 2, 4, 5, 7, 8\}$, $a \times b \bmod 9$
   (f) $\mathbb{Q}$, $(a + b)/2$

3. For each corresponding set and operator in Question 2, find the inverses of the given elements; if an element has no inverse in the given set under the given operator, state that.

   (a) 2, 5, 12, 1   no inverses
   (b) 4, -3, 0, 1
   (c) 0, 3, 6, 9
   (d) -1, 0, 1, 2
   (e) 1, 5, 7, 8
   (f) 1, 2, 3, 4

4. For each shape below, (a) write down its rotational symmetries, and (b) draw lines indicating its mirror symmetries. The first example is solved.



rotations: $\{0°, 120°, 240°\}$

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Assignment 11, part 2

5. **Commutativity.** A square has four rotational symmetries (0°, 90°, 180°, and 270°), and four mirror symmetries. These eight symmetries constitute a group under the binary operation of performing one transformation after the other; this group is sometimes called D_4.

   - Find two elements a, b in D_4 so that $a \star b = b \star a$.
   - Find two elements a, b in D_4 so that $a \star b \neq b \star a$.

6. **Experimental mathematics.** Two important classes of symmetry groups are the cyclic groups and dihedral groups. If a shape has n distinct rotations (including the rotation by 0 degrees) and no other symmetries, then its associated symmetry group is known as the **cyclic group** C_n. If a shape has n distinct rotations AND n distinct mirrors, then the associated group is known as the **dihedral group** D_n.

   

   Almost all tire rims have associated symmetry groups C_n or D_n (for example C_5 and D_7 for the examples above). Look at 25 cars and record the associated symmetry group of the rims of each. Tabulate your results; list symmetry groups in order of decreasing popularity.

7. In three dimensions, shapes can have multiple axes of rotation. The **order** of a rotational axis is the number of rotational symmetries about that axis. Consider for example a line passing through two opposite corners of a cube; its order is 3, since the cube can be rotated 0°, 120°, or 240° about that axis.

   Think about each of the five platonic solids. Each have multiple axes of different orders. For example, the tetrahedron has 3 rotational axes of order 2, and 4 rotational axes of order 3. Think about this. Now determine the number of rotational axes of each order, for the remaining four platonic solids. All orders are between 2 and 5. This problem will require visualization and thinking.