ASSIGNMENT 5
SOLUTIONS

1a: $1+2+\ldots+n=\sum_{i=1}^{n} i$

b: $1-2+3-4\ldots n=\sum_{i=1}^{n}(-1)^{i+1} i$

c: $x+x^2+\ldots+x^n=\sum_{i=1}^{n} x^i$

d: $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}=\sum_{i=1}^{n}\frac{1}{i}$

e: $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\frac{1}{n}=\sum_{i=1}^{n}\frac{(-1)^{i+1}}{i}$

f: $x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}=\sum_{k=1}^{n}\frac{x^k}{k!}$

2: a $\sum_{i=1}^{2} i=1+2=3$

b: $\sum_{i=1}^{3}(-1)^{i+1} i=1-2+3=2$

c: $\sum_{i=1}^{4} x^i = x + x^2 + x^3 + x^4$

d: $\sum_{i=1}^{5} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{60}{60} + \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60}$

$= \frac{137}{60}$

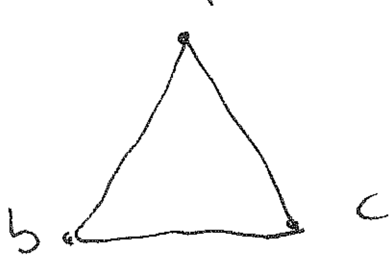
e: $\sum_{k=1}^{6} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = \frac{60}{60} - \frac{30}{60} + \frac{20}{60} - \frac{15}{60} + \frac{12}{60} - \frac{10}{60}$

$= \frac{30}{60} + \frac{5}{60} + \frac{2}{60}$

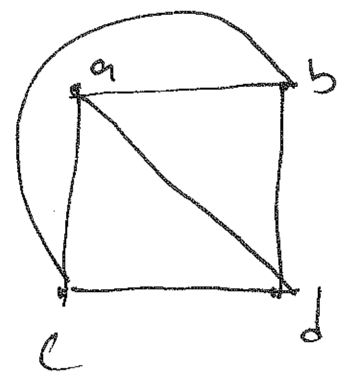
$= \frac{37}{60}$

f: $\sum_{k=1}^{7} \frac{x^k}{k!} = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!}$

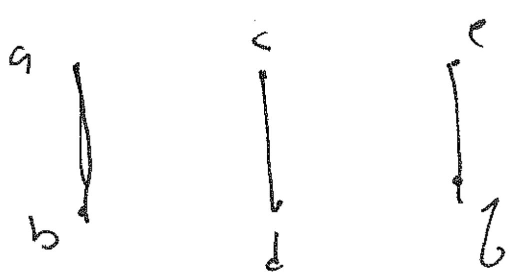
3: a: $V_1 = \{a, b, c\}$ $\quad E_1 = \{\{a, b\}, \{c, b\}, \{a, c\}\}$



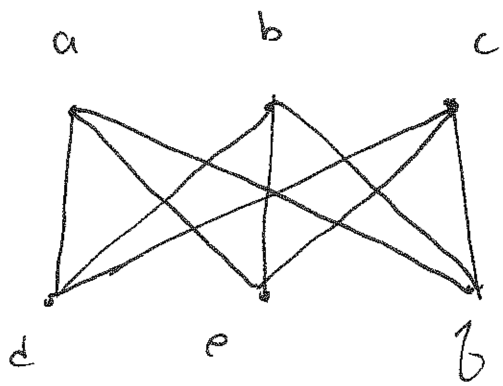
b: $V_2 = \{a, b, c, d\}$ $\quad E_2 = \{\{a,b\}, \{b,c\}, \{c,d\}, \{d,a\},$
$\{a,c\}, \{b,d\}\}$



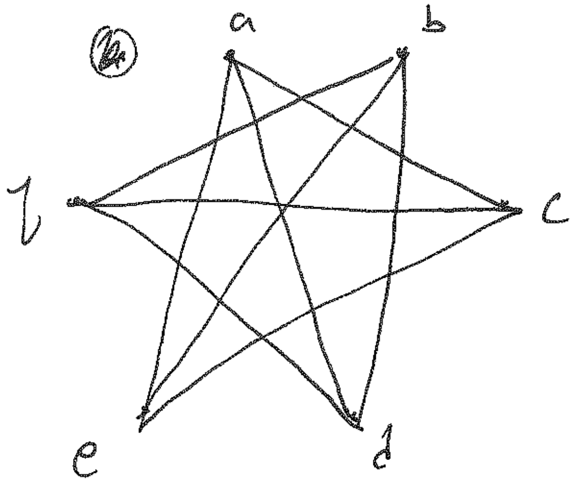
c: $V_3 = \{a, b, c, d, e, f\}$ $\quad E_3 = \{\{a,b\}, \{c,d\}, \{e,f\}\}$



d: $V_4 = \{a, b, c, d, e, f\}$ $\quad E_4 = \{\{a,d\}, \{a,e\}, \{a,f\}, \{b,d\}, \{b,e\}$
$\{b,f\}, \{c,d\}, \{c,e\}, \{c,f\}\}$

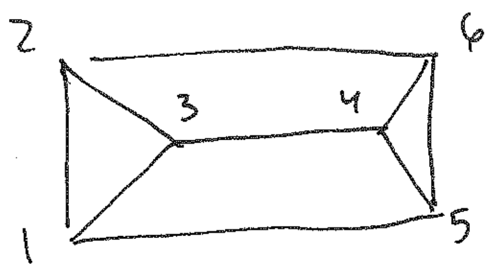


4:



$V = \{a, b, c, d, e, f\}$

$E \{ \{a,c\}, \{a,d\}, \{a,e\}, \{b,d\}, \{b,e\}, \{b,f\}, \{c,f\}, \{c,e\}, \{d,f\} \}$



$V = \{1, 2, 3, 4, 5, 6\}$

$E = \{ \{1,2\}, \{1,3\}, \{1,5\}, \{2,3\}, \{2,6\}, \{3,4\}, \{4,5\}, \{4,6\}, \{5,6\} \}$

5: LET $A \subset V$ BE THE SET OF VERICES WITH EVEN DEGREE. $B \subset V$ BE THE SET OF VERTICES OF ODD DEGREE. NOTE $A \cap B = \emptyset$ AND $A \cup B = V$.

THEN $2|E| = \sum_{i=1} \deg v_i = \sum_{v_i \in A} \deg v_i + \sum_{v_k \in B} \deg v_k$

SO $$2|E| - \sum_{v_i \in A} \deg v_i = \sum_{v_k \in B} \deg v_k$$

THE LEFT HAND SIDE IS EVEN SINCE ITS A SUM OF EVEN NUMBERS.

$\Longrightarrow$ THE RIGHT HAND SIDE IS EVEN. SINCE THE SUM OF AN ODD NUMBER OF ODD NUMBERS IS ODD, $|B|$ MUST BE EVEN.

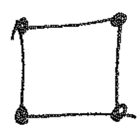
6: NOTE THAT ON A GRAPH WITH n VERTICES, THE DEGREE OF ANY VERTEX IS AT MOST $n-1$.

SO $$2|E| = \sum_{i=1}^{n} \deg v_i \le \sum_{i=1}^{n} n-1 = n(n-1)$$

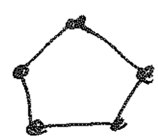
$$\Longrightarrow |E| \le \frac{n(n-1)}{2} = \frac{n^2-n}{2}$$

7: $n=3$:

$n=4$

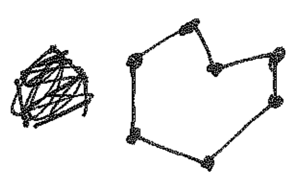


$n=5$

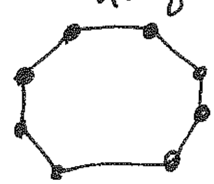


$n=6$

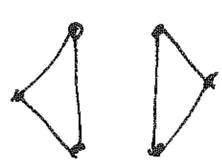
$n=7$



$n=8$



8: HERE IS A NON-CONNECTED 2-REGULAR GRAPH:



9: a, b, c ARE PLANAR, WHICH IS CLEAR FROM THE DRAWINGS.

d IS $K_{3,3}$ AND SO ITS NOT PLANAR.

SEE SOLUTIONS TO ASSIGNMENT 6 FOR DETAILS.