Assignment

6 solutions

1: a - 1  b - 3  c - 2  d - 4

This induces the map:
\{a, d\} \rightarrow \{1, 4\}
\{a, c\} \rightarrow \{1, 2\}
\{c, d\} \rightarrow \{2, 4\}
\{b, c\} \rightarrow \{3, 2\}

2: a - 1  b - 8  c - 3  d - 6  g - 7  h - 2
i - 5  j - 4

3: Not isomorphic. Left graph is connected, right one is not.

4: a - 2  b - 5  c - 3  d - 6  e - 1  g - 4

Note: there are other acceptable isomorphisms.

5: A graph with no edges is bipartite, so 0 zero is the smallest possible value of e. To each element v of A, there are |B| possible edges that connect to v. (said differently: \deg v \leq |B|) \implies |e| \leq |A||B|. Equality is possible: connect each element of A to every element of B.
(The largest possible value is $|A|/|B|$)

6. **First note** that the **isomorphism** given in 2, the graph associated to a tree is **bipartite**.

b. **First note** that the complete graph on 3 vertices (visually: $\triangle$) is **not** bipartite.

Next note that any planar graph with a degree 3 face contains a copy of $K_3$.

Since any subgraph of a bipartite graph is **bipartite**, $K_3$ is not a subgraph of any bipartite graph. This is the same as saying a bipartite graph has no faces of degree 3. The degree of any face is $\geq 3$. Since 3 is not possible for a 3-regular planar bipartite graph, the degree of any face is $\geq 4$.

7. i. Since $\sum_{F \in F} \deg F_i = 2E$, $K_{3,3}$ has 6 vertices and 9 edges. If it were planar, by Euler's formula, it would have five faces. The inequality above then gives $4(5) \leq 2(9)$ $\Rightarrow 20 \leq 18$. 
This is a contradiction $\Rightarrow K_{3,3}$ is not planar.

8: It's planar:

9: This is the same proof as #5 in Assignment 5, replacing vertices with faces.