ASSIGNMENT 6 SOLUTIONS

1: a - 1 b - 3 c - 2 d - 4

THIS INDUCES THE MAP
$\{a,d\} - \{1,4\}$ $\{a,c\} - \{1,2\}$ $\{c,d\} - \{2,4\}$
$\{b,c\} - \{3,2\}$

2: a - 1 b - 8 c - 3 d - 6 g - 7 h - 2
i - 5 j - 4

3: NOT ISOMORPHIC. LEFT GRAPH IS CONNECTED, RIGHT ONE IS NOT.

4: a - 2 b - 5 c - 3 d - 6 e - 1 f - 4

NOTE: ~~THERE~~ FOR 1, 2, 4, THERE ARE OTHER ACCEPTABLE ISOMORPHISMS.

5: A GRAPH WITH NO EDGES IS BIPARTITE, SO ZERO IS THE SMALLEST POSSIBLE VALUE OF E. TO EACH ELEMENT v OF A, THERE ARE $|B|$ POSSIBLE EDGES THAT CONNECT TO v. (SAID DIFFERENTLY: $\deg v \leq |B|$) $\Rightarrow |E| \leq |A||B|$.
EQUALITY IS POSSIBLE: CONNECT EACH ELEMENT OF A TO EVERY ELEMENT OF B.

(THE LARGEST POSSIBLE VALUE IS $|A||B|$)

6 a: ~~FIRST NOTE~~ BY THE ISOMORPHISM GIVEN IN 2, THE GRAPH ASSOCIATED TO A CUBE IS BIPARTITE.

b: FIRST NOTE THAT THE COMPLETE GRAPH ON 3 VERTICES (VISUALLY: ) IS NOT BIPARTITE.

NEXT NOTE THAT ANY PLANAR GRAPH WITH A DEGREE 3 FACE CONTAINS A COPY OF K_3.

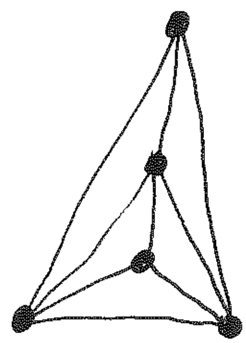
SINCE ANY SUBGRAPH OF A BIPARTITE GRAPH IS BIPARTITE, K_3 IS NOT A SUBGRAPH OF ANY BIPARTITE GRAPH. THIS IS THE SAME AS SAYING A BIPARTITE GRAPH HAS NO FACES OF DEGREE 3. THE DEGREE OF ANY FACE IS ≥ 3. SINCE 3 IS NOT POSSIBLE FOR A PLANAR BIPARTITE GRAPH, THE DEGREE OF ANY FACE IS ≥ 4.

7: SINCE $\sum_{i=1}^{F} \deg F_i = 2E$, QUESTION 6 GIVES THAT FOR ANY PLANAR BIPARTITE GRAPH $4F \leq 2E$. $K_{3,3}$ HAS 6 VERTICES AND 9 EDGES. IF IT WERE PLANAR, BY EULER'S FORMULA, IT WOULD HAVE FIVE FACES. THE INEQUALITY ABOVE THEN GIVES $4(5) \leq 2(9)$

$20 \leq 18$.

THIS IS A CONTRADICTON. $\Longrightarrow K_{3,3}$ IS NOT PLANAR.

8: ITS PLANAR:



9: THIS IS THE SAME PROOF AS #5 IN ASSIGNMENT 5, REPLACING VERITICES WITH FACES.