SOLUTIONS TO ASSIGNMENT 7

1a: TETRAHEDRON, 4 VERTICES, 6 EDGES, 4 FACES, $\deg v_i = 3$, $\deg f_i = 3$

b: CUBE, 8 VERTICES, 12 EDGES, 6 FACES $\deg v_i = 3$ $\deg f_i = 4$

c: OCTAHEDRON, 6 VERTICES, 12 EDGES, 8 FACES $\deg v_i = 4$ $\deg f_i = 3$

d: DODECAHEDRON, 20 VERTICES, 30 EDGES, 12 FACES, $\deg v_i = 3$ $\deg f_i = 5$

e: ICOSAHEDRON, 12 VERTICES, 30 EDGES 20 FACES $\deg v_i = 5$ $\deg f_i = 3$.

2: THIS PROCESS DECRIBES TAKING THE DUAL OF THE PLATONIC SOLIDS.

- THE DUAL OF THE TETRAHEDRON IS THE TETRAHERON
- THE DUAL OF THE CUBE IS THE OCTAHEDRON
- THE DUAL OF THE OCTAHEDRON IS THE CUBE
- THE DUAL OF THE DODECAHEDRON IS THE ICOSAHEDRON
- THE DUAL OF THE ICOSAHEDRON IS THE DODECAHEDRON.

3: THREE FACES MEETING AT EVERY CORNER MEANS THE DEGREE OF EACH VERTEX IS THREE. WE'VE ALREADY ~~PROO~~ PROVED THAT THE NUMBER OF VERTICES OF ODD DEGRE IS EVEN FOR ANY GRAPH.

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SINCE EVERY VERTEX OF THE GRAPH OF A SIMPLE POLYHEDRON HAS ~~DEGREE~~ ODD DEGREE ~~[illegible]~~ (3), THERE MUST BE AN EVEN NUMBER OF VERTICES.

4: - A GRAPH WITH n VERTICES CAN HAVE n VERTICES OF DEGREE ZERO. (TAKE A GRAPH WITH NO EDGES)

- A GRAPH WITH n VERTICES CAN NOT HAVE ANY VERTICES OF DEGREE n. ~~TH~~ FOR ANY VERTEX, THERE ARE ONLY n-1 VERTICES TO CONNECT TO.

- IF A GRAPH HAS A VERTEX OF DEGREE n-1, THEN THERES AN EDGE BETWEEN THAT VERTEX AND EVERY OTHER VERTEX. SO EVERY VERTEX IS CONNECTED TO AT LEAST ONE OTHER VERTEX $\Longrightarrow$ THERE ARE NO VERTICES OF DEGREE ZERO.

5: IN A GRAPH WITH n VERTICES, IF EACH VERTEX WERE TO HAVE A DIFFERENT DEGREE, THERE WOULD BE ONE VERTEX OF DEGREE $n-1$, ONE OF DEGREE $n-2$, ONE OF DEGREE $n-3$, . . . , ONE OF DEGREE 1, AND ONE OF DEGREE ZERO. THIS IS IMPOSSIBLE BY QUESTION 4.

6: FIRST NOTE THAT THE FORMULA HOLDS WHEN $n=1$:

$$\frac{1(1+1)(2\cdot 1+1)}{6} = \frac{2\cdot 3}{6} = 1 = 1^2$$

NOW ASSUME THE FORMULA HOLDS FOR SOME $n \in \mathbb{N}$, WE NEED TO SHOW IT ALSO HOLDS FOR $n+1$:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6}$$

$$= \frac{(n+1)\big(n(2n+1) + 6(n+1)\big)}{6}$$

$$= \frac{(n+1)(2n^2 + 7n + 6)}{6}$$

$$= \frac{(n+1)(n+2)(2n+3)}{6} = \cancel{(n+1)(n+1+1}$$

$$= \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6}$$