ASSIGNMENT #8 SOLUTIONS

1: $\cancel{2}\,2\cdot 3+1 = 7 \Longrightarrow 7/3$ GIVES 2 r 1

$2\cdot 9+1 = 19 \Longrightarrow 19/2$ GIVES 9 r 1

$3\cdot 19+18 = 57+18 = 75 \Longrightarrow 75/19$ GIVES 3 r 18

$17\cdot 17+4 = 189+4 = 293 \Longrightarrow \cancel{288}\; 293/17$ GIVES 17 r 4

$23\cdot 7+22 = 161+22 = 183 \Longrightarrow 183/23$ GIVES 7 r 22

$239\cdot 37+68 = 8843+68 = 8911 \Longrightarrow 8911/239$ GIVES 37 r 68

$21\cdot 1560+8 = 32760+8 = 32768 = 2^{15} \Longrightarrow 2^{15}/21$ GIVES 1560 r 8

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      526315
19 ) 10000000
      95
       50
       38
       120
       114
         60
         57
          30
          19
          110
           95
           15
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$\Longrightarrow 10^7/19$ GIVES

526315 r 15

2: $19 = 13 \cdot 1 + 6 \implies \boxed{19 \equiv 6 \bmod 13}$

$-5 = -2 \cdot 4 + 3 \implies \boxed{-5 \equiv 3 \bmod 4}$

$100 = 4 \cdot 25$, SO 1500 IS A MULTIPLE OF 4

$40 = 4 \cdot 10$ SO $1500 + 40 = 1540$ IS A MULTIPLE OF 4. $1557 = 1540 + 17$. $17 = 4 \cdot 4 + 1$

$\implies \boxed{1557 \equiv 1 \bmod 4.}$

$-21 = -2 \cdot 11 + 1 \implies \boxed{-21 \equiv 1 \bmod 11}$

$23 = 1 \cdot 13 + 10 \implies \boxed{23 \equiv 10 \bmod 13}$

$4^2 = 16 \equiv -1 \bmod 17$, so

$$\begin{aligned} 14^{13} &= 4^2 \cdot 4^2 \cdot 4^2 \cdot 4^2 \cdot 4^2 \cdot 4^2 \cdot 4 \\ &\equiv (-1)(-1)(-1)(-1)(-1)(-1) \cdot 4 \bmod 17 \\ &\equiv 1 \cdot 1 \cdot 1 \cdot 4 \bmod 17 \\ &\equiv \boxed{4 \bmod 17} \end{aligned}$$

$17 \equiv 2 \bmod 15$ so

$$\begin{aligned} 17^2 &\equiv 4 \bmod 15 \\ 17^3 &\equiv 8 \bmod 15 \\ 17^4 &\equiv 1 \bmod 15 \\ 17^5 &\equiv 2 \bmod 15 \\ &\boxed{17^6 \equiv 4 \bmod 15} \end{aligned}$$

$$17 \equiv 1 \bmod 15 \implies 17^{357} \equiv (1)^{357} \bmod 16$$
$$\equiv \boxed{1 \bmod 16}$$

3:

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

4: LET n BE AN EVEN INTEGER. THEN $n = 2k$ FOR SOME INTEGER k AND

$$n^2 = (2k)^2 = 4k^2 \Longrightarrow n^2 \equiv 0 \bmod 4$$

5: LET n BE AN ODD INTEGER. THEN $n = 2j+1$ FOR SOME INTEGER j AND

$$n^2 = (2j+1)^2 = 4j^2 + 4j + 1$$

$$= 4(j^2+j)+1 \Longrightarrow n^2 \equiv 1 \bmod 4$$

6: - IF a AND b ARE BOTH EVEN, THEN

$$a^2+b^2 \equiv 0+0 \bmod 4 \equiv 0 \bmod 4.$$

- IF a IS EVEN AND b IS ODD, OR IF a IS ODD AND b IS EVEN, THEN $a^2+b^2 \equiv 1+0 \bmod 4$

$$\equiv 1 \bmod 4$$

- IF a AND b ARE BOTH ODD THEN

$$a^2+b^2 \equiv 1+1 \bmod 4 \equiv 2 \bmod 4.$$

$\Longrightarrow$ THERE ARE NO INTEGERS a, b SUCH THAT $a^2+b^2 \equiv 3 \bmod 4$

7: LET a_m BE THE DIGITS OF SOME INTEGER n.

THEN $n = a_1 + 10 \cdot a_2 + 100 \cdot a_3 + \ldots + 10^{m-1} a_{m-1} + 10^m a_m$.

(SO, FOR EXAMPLE, IF $n = 253$ $a_1 = 3$ $a_2 = 5$ AND $a_3 = 3$)

NOTE $10 \equiv 1 \bmod 3$.

THEN $n \equiv (a_1 + 10 \cdot a_2 + \ldots + 10^{m-1} a_{m-1} + 10^m a_m) \bmod 3$

$\equiv a_1 \bmod 3 + (10 \cdot a_2) \bmod 3 + (100 \cdot a_3) \bmod 3 \ldots$
$\ldots + (10^{m-1} a_{m-1}) \bmod 3 + (10^m a_m) \bmod 3$

$\equiv a_1 \bmod 3 + a_2 \bmod 3 + \ldots + a_{m-1} \bmod 3 + a_m \bmod 3$

$\equiv (a_1 + a_2 + a_3 + \ldots + a_{m-1} + a_m) \bmod 3$

SO $n \equiv 0 \bmod 3$ IS EQUIVALENT TO $(a_1 + a_2 \ldots + a_m) \equiv 0 \bmod 3$.

IN OTHER WORDS n IS A MULTIPLE OF 3 IF AND ONLY IF THE SUM OF ITS DIGITS IS A MULTIPLE OF 3.

8: THIS QUESTION DOES NOT REQUIRE YOU TO FILL OUT THE MULTIPLICATION TABLES, BUT THEY GIVE ALL THE RELEVANT INFORMATION AND ARE ~~INSTRUCTIVE~~ INSTRUCTIVE. HERE THEY ARE.

MULTIPLICATION MOD 6

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

LOOKING AT THE DIAGONAL, ZERO HAS ONE SQUARE ROOT: ZERO. 3 HAS ONE SQUARE ROOT: 3. 4 HAS TWO SQUARE ROOTS: 2 AND 4. 1 HAS ~~2~~ TWO SQUARE ROOTS: 1 AND 5. 5 AND 2 HAVE NO SQUARE ROOTS.

MULTIPLICATION MOD 9

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8
2	0	2	4	6	8	1	3	5	7
3	0	3	6	0	3	6	0	3	6
4	0	4	8	3	7	2	6	1	5
5	0	5	1	6	2	7	3	8	4
6	0	6	3	0	6	3	0	6	3
7	0	7	5	3	1	8	6	4	2
8	0	8	7	6	5	4	3	2	1

ZERO HAS 3 SQUARE ROOTS: 0, 3, AND 6.

1 HAS TWO SQUARE ROOTS: 1 AND 8

4 HAS TWO SQUARE ROOTS: 2 AND 7

7 HAS TWO SQUARE ROOTS: 4 AND 5.

2, 3, 5, 6, 8 HAVE NO SQUARE ROOTS.

MULTIPLICATION MOD 12

	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11
2	0	2	4	6	8	10	0	2	4	6	8	10
3	0	3	6	9	0	3	6	9	0	3	6	9
4	0	4	8	0	4	8	0	4	8	0	4	8
5	0	5	10	3	8	1	6	11	4	9	2	7
6	0	6	0	6	0	6	0	6	0	6	0	6
7	0	7	2	9	4	11	6	1	8	3	10	5
8	0	8	4	0	8	4	0	8	4	0	8	4
9	0	9	6	3	0	9	6	3	0	9	6	3
10	0	10	8	6	4	2	0	10	8	6	4	2
11	0	11	10	9	8	7	6	5	4	3	2	1

ZERO HAS TWO SQUARE ROOTS: ZERO AND 6

1 HAS FOUR SQUARE ROOTS: 1, 5, 7, AND 11

4 HAS FOUR SQUARE ROOTS: 2, 4, 8 AND 10

9 HAS TWO SQUARE ROOTS: 3 AND 9.

2, 3, 5, 6, 7, 8, 10, 11 HAVE NO SQUARE ROOTS.