

ASSIGNMENT #11 SOLUTIONS

a: THE PRODUCT OF 2 NATURAL NUMBERS IS A NATURAL NUMBER

⇒ CLOSED

b: THE SUM OF 2 INTEGERS IS AN INTEGER ⇒ CLOSED

c: ADDITION MOD 12:

	0	3	6	9
0	0	3	6	9
3	3	6	9	0
6	6	9	0	3
9	9	0	3	6

⇒ CLOSED

d: $2+2=4$ ⇒ NOT CLOSED

e. MULTIPLICATION MOD 9:

	1	2	4	5	7	8
1	1	2	4	5	7	8
2	2	4	8	1	5	7
4	4	8	7	2	1	5
5	5	1	2	7	8	4
7	7	5	1	8	4	2
8	8	7	5	4	2	1

⇒ CLOSED

f. THE SUM OF 2 RATIONAL NUMBERS IS RATIONAL AND DIVIDING A RATIONAL NUMBER BY 2 RETURNS A RATIONAL NUMBER

⇒ CLOSED

2a: FOR ANY NATURAL NUMBER a ,

$a \cdot 1 = 1 \cdot a = a \implies$ 1 IS THE IDENTITY

b: FOR ANY INTEGER, a , $0 + a = a + 0 = a$

\implies 0 IS THE IDENTITY

c: BY THE ADDITION TABLE FROM #1,

0 IS THE IDENTITY

d: 0 IS THE IDENTITY FOR THE SAME REASON AS b.

e: BY THE MULTIPLICATION TABLE FROM #1, 1 IS THE IDENTITY

f: NO IDENTITY EXISTS IF AN IDENTITY

e EXISTED, THEN $\frac{0+e}{2} = 0$

SO $e=0$. HOWEVER $\frac{1+e}{2} = 1$

SO $e=1 \implies$ (CONTRADICTION)

3a: 2, 5, 12 HAVE NO INVERSES. 1 IS ITS OWN INVERSE

b: THE INVERSE OF 4 IS -4
 " " -3 IS 3
 " " 0 IS 0
 " " 1 IS -1

c: BY THE ADDITION TABLE FROM #1,

THE INVERSE OF 0 IS 0
 " " 3 IS 9
 " " 6 IS 6
 " " 9 IS 3

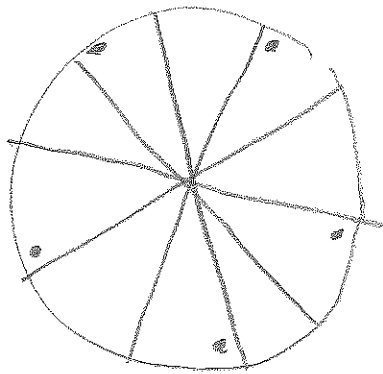
d: THE INVERSE OF -1 IS 1
 " " 0 IS 0
 " " 1 IS -1
 " " 2 IS -2

e: BY THE MULTIPLICATION TABLE FROM #1

THE	INVERSE	OF	1	IS	1
"	"	"	2	IS	5
"	"	"	7	IS	4
"	"	"	8	IS	8

f: THERE'S NO IDENTITY ELEMENT, SO TALKING ABOUT INVERSES MAKES NO SENSE.

4:



ROTATIONS: $\{0^\circ, 72^\circ, 144^\circ, 216^\circ, 288^\circ\}$

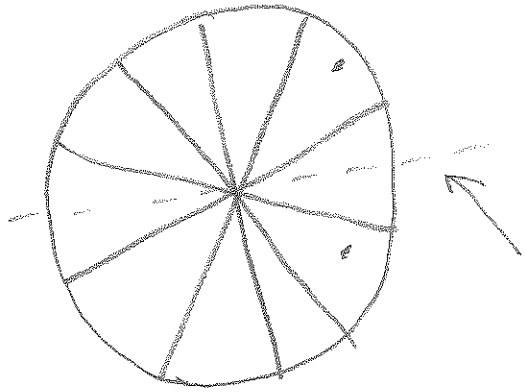
NO MIRROR SYMMETRIES.

NO ROTATIONAL SYMMETRIES

(OR, ROTATIONAL SYMMETRIES: $\{0^\circ\}$)

ONE MIRROR

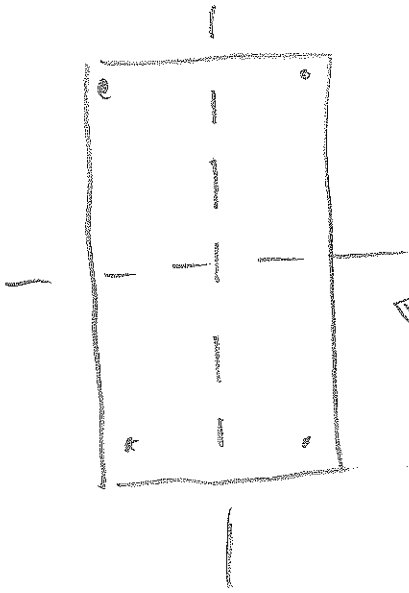
SYMMETRY



ROTATIONAL SYMMETRIES:

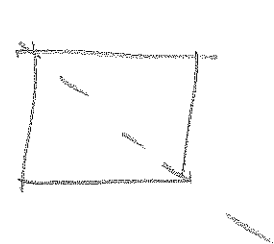
$\{0^\circ, 180^\circ\}$

2 MIRROR SYMMETRIES

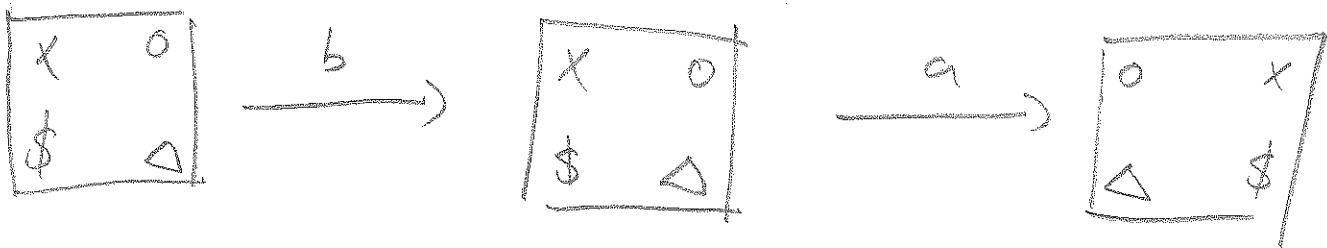
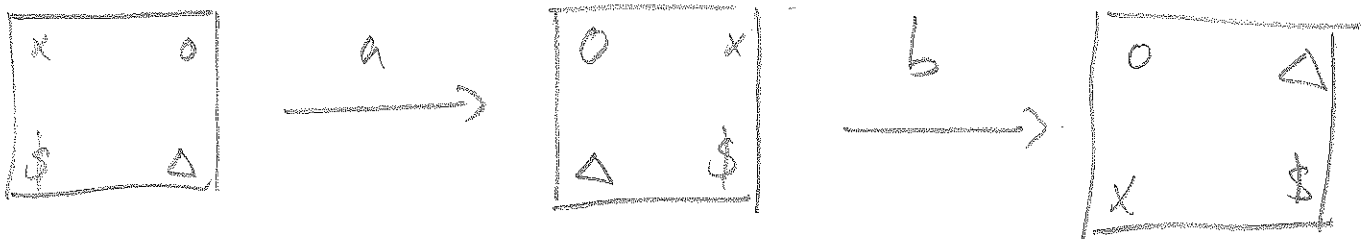


5: LET a BE ROTATION BY 0° (THE IDENTITY ELEMENT) AND LET b BE ROTATION BY 90° . THEN $a * b = b * a = b$ (NOTE, THERE ARE OTHER POSSIBLE ANSWERS.

LET a BE REFLECTION ABOUT THE VERTICAL AXIS AND LET b BE REFLECTION ABOUT THE INDICATED DIAGONAL AXIS:



THEN



So $a \circ b$ IS ROTATION BY 270°
 AND $b \circ a$ IS ROTATION BY 90° (MY ROTATIONS ARE CLOCKWISE) $\implies a \circ b \neq b \circ a$.

6: ANSWERS VARY.

7: CUBE - AXES THROUGH THE CENTERS OF OPPOSITE FACES GIVE

3 ROTATIONAL AXES OF ORDER 4.

- AXES JOINING OPPOSITE

VERTICES GIVE 4 ROTATIONAL AXES OF ORDER 3.

- AXES JOINING THE MIDPOINTS OF OPPOSITE EDGES GIVE

6 ROTATIONAL AXES OF ORDER 2

OCTAHEDRON - THIS IS DUAL TO THE CUBE, REPLACE "FACE" WITH "VERTEX" AND "VERTEX" WITH "FACE" AND YOU'RE DONE.

DODECAHEDRON

- AXES THROUGH THE CENTERS OF OPPOSITE

FACES GIVE 6 ROTATIONAL

AXES OF ORDER 5

- AXES THROUGH OPPOSITE VERTICES GIVE

10 ROTATIONAL

AXES OF ORDER 3

- AXES THROUGH THE MIDPOINTS OF OPPOSITE EDGES GIVE

15 ROTATIONAL AXES OF

ORDER 3

ICOSAHEDRON - DUAL TO THE DODECAHEDRON, SWITCH FACES AND VERTICES ABOVE.