MATH 360 — HOMEWORK 12.

due on Wednesday, November 30.

by J. E. Marsden and M. J. Hoffman

Additional Reading: “Foundations of Modern Analysis”
by J. Dieudonné

Topics:
• Continuous Mappings
  – 4.6 Uniform Continuity
  – 4.7 Differentiation of Functions of One Variable
• Bounded Multilinear Maps

Twelvth Homework Assignment.

Reading:
• Read Section 4.7. Read your notes.
• You can find an even more general form (than we proved in class) of the Mean Value Theorem in Dieudonné (8.5.2 and 8.5.1).
• You can find some of the facts about multilinear maps in the Worked Examples for Chapter 4. You can find all the facts in Dieudonné (5.5 and 5.7).

The Second Midterm: Covers everything up to and including Section 4.7. This includes everything we did in class and everything you had to read in Dieudonné, including the above Reading assignment. Pay close attention to the Examples in our textbook, both in the text and at the end of each Chapter. Look back at the homework problems.

Exercises: Here are a few sample problems for the Midterm, that did not make it into past home-works (you do not have to hand them in):

Problem 1. Suppose \( f : [-1, 1] \to \mathbb{R} \) is continuous on the closed interval \([-1, 1]\), and twice differentiable on the open interval \((-1, 1)\). Suppose also that \( f(-1) = 1, f(0) = -1 \) and \( f(1) = -1 \).

Prove that there exists \( c \in (-1, 1) \) such that \( f''(c) = 2 \).
Problem 2. Let \((E, \| \cdot \|_\infty)\) be the Normed Vector Space consisting of all sequences
\[ \{ c = \{c_n\}_{n \in \mathbb{N}} \mid c_n \in \mathbb{R}, \text{ only a finite number of them are } \neq 0 \} \]
with the sup-norm
\[ \|c\|_\infty = \sup_{n \in \mathbb{N}} |c_n|. \]
Prove that the bilinear map \(\phi(c, d) = \sum_{n \in \mathbb{N}} c_n d_n\) is not continuous.

Problem 3. Let \(E\) be the same vector space as above, but endowed with the 2-norm
\[ \|c\|_2 = \sqrt{\sum_{n \in \mathbb{N}} |c_n|^2}. \]
Prove that the bilinear map \(\phi(c, d) = \sum_{n \in \mathbb{N}} c_n d_n\) is continuous.