MATH 360 — HOMEWORK 3.

due on Friday, September 20.

by J. E. Marsden and M. J. Hoffman

Additional Reading: “Foundations of Modern Analysis”
by J. Dieudonné

Topics:
• Introduction: Sets and Functions
• 1 The real Line and Euclidian Space
  – 1.1 Ordered Fields and the Number System
  – 1.2 Completeness and the Real Number System

Third Homework Assignment.

Reading:
• Read Section 1.2 paying attention to all the examples.

Exercises:

Problem 1. Prove that if \( \lim_{n \to \infty} x_n = x \) and \( \lim_{n \to \infty} y_n = y \), then
\[
\lim_{n \to \infty} x_n y_n = xy.
\]

Problem 2. Prove that the sequence \( \left\{ \frac{1}{n} \right\}_{n \in \mathbb{N}} \) converges in the ordered field
\( F \) if and only if \( F \) is Archimedean, and then it converges to 0.

Problem 3. Prove that any complex number \( z \in \mathbb{C} \) is a square, that is there
exists \( w \in \mathbb{C} \), such that \( z = w^2 \). Conclude that there is no order relation on
\( \mathbb{C} \), to turn it into an ordered field.

Problems:
• Page 35: problems: 3, 4, 5
• Page 45: problems: 1, 2, 3, 4, 5

The topics and page numbers are from the textbook.