MATH 360 — HOMEWORK 6.

due on Friday, October 18

by J. E. Marsden and M. J. Hoffman

Additional Reading: “Foundations of Modern Analysis”
by J. Dieudonné

Topics:
• The Topology of Euclidian Space
  – 2.4 Accumulation Points
  – 2.5 Closure of a Set
  – 2.6 Boundary of a set
  – 2.7 Sequences
  – 2.8 Completeness

Sixth Homework Assignment.

Reading:
• Read Sections 2.4, 2.7 and 2.8 paying attention to all the examples.

Exercises:

Problem 1. Prove that if $1 \leq p \leq q \leq \infty$, then

$$\|x\|_q \leq \|x\|_p,$$

for every $x \in \mathbb{R}^N$.

(Hint: It is enough to consider the case $\|x\|_p = 1$.)

Problem 2. Prove that if a Cauchy sequence $\{x_n\}_{n \in \mathbb{N}}$, $x_n \in (M, d)$, has a convergent subsequence, than the sequence itself is convergent.

• Page 115: problems: 2, 3, 4, 6
• Page 120: problems: 4
• Page 125: problems: 2, 5

The topics and page numbers are from the textbook.