MATH 360 — HOMEWORK 7.

due on Friday, October 25

by J. E. Marsden and M. J. Hoffman

Additional Reading: “Foundations of Modern Analysis”
by J. Dieudonné

Topics:
• The Topology of Euclidean Space
  – 2.9 Series of Real Numbers and Vectors
• Compact and Connected Sets
  – 3.1 Compactness

Seventh Homework Assignment.

Reading:
• Read Sections 2.9 and 3.1 paying attention to all the examples. Read your notes.

Exercises:

Problem 1. Prove that if $d_1$ and $d_2$ are equivalent metrics on $M$ and $T$ is a subset of $M$, then $T$ is totally bounded in $(M, d_1)$ if and only if it is totally bounded in $(M, d_2)$.

Problem 2. Prove that a subset $T$ in the metric space $(M, d)$ is totally bounded if and only if for every $\varepsilon > 0$ there exist $x_1, \ldots, x_N \in T$ such that

$$T \subset \bigcup_{i=1}^{N} D(x_i, \varepsilon).$$

(The important difference is that the centers of the discs are in $T$.)

Problem 3. Prove that if $T$ is totally bounded in $(M, d)$, then $\overline{T}$ (the closure of $T$) is totally bounded too.

• Page 149: problems: 52, 53
• Page 155: problems: 1, 4
• Page 172: problems: 5, 7, 30

The topics and page numbers are from the textbook.