MATH 360 — HOMEWORK 8.

due on Friday, November 1

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Additional Reading: “Foundations of Modern Analysis”
by J. Dieudonné

Topics:
• Compact and Connected Sets
  – 3.1 Compactness
  – 3.2 The Heine-Borel Theorem
  – 3.3 Nested Set Property
  – 3.5 Connected Sets
• Continuous Mappings
  – 4.1 Continuity

Eighth Homework Assignment.

Reading:
• Read Sections 3.2, 3.3, 3.5 and 4.1 paying attention to all the exam-
ples. Read your notes.

Exercises:

Problem 1. Suppose $A \subset M$ is a subset of the metric space $(M, d)$. Prove
that $U \subset A$ is an open set in the metric space $(A, d|_A)$ if and only if there
exists an open set $W \subset M$ in the metric space $(M, d)$, such that

$$U = W \cap A.$$

Problem 2. Prove that a subset $I \subset \mathbb{R}$ is an interval if and only if it has
the property that $[x, y] \subset I$, for every $x, y \in I$, $x \leq y$.

Problem 3. Prove that if $x$ is any point in the metric space $(M, d)$, then
the union of all connected sets that contain $x$ is connected and contains $x$.
(It is called the connected component of $x$.)

• Page 159: problems: 1, 3, 4
• Page 164: problems: 1, 2, 4
• Page 172: problems: 1, 9, 11

The topics and page numbers are from the textbook.