MATH 360 — HOMEWORK 8.

due on Friday, October 28.

by J. E. Marsden and M. J. Hoffman

Additional Reading: “Foundations of Modern Analysis”
by J. Dieudonné

Topics:
• Compact and Connected Sets
  – 3.1 Compactness
  – 3.2 The Heine-Borel Theorem
  – 3.3 Nested Set Property

Eighth Homework Assignment.

Reading:
• Read Sections 3.1, 3.2 and 3.3. Read your notes.

Exercises:

Problem 1. Prove that if $d_1$ and $d_2$ are equivalent metrics on $M$ and $T$ is a subset of $M$, then $T$ is totally bounded in $(M, d_1)$ if and only if it is totally bounded in $(M, d_2)$.

Problem 2. Prove that if $T$ is totally bounded in $(M, d)$, then $\overline{T}$ (the closure of $T$) is totally bounded too.

Problem 3. Suppose $A \subset M$ is a subset of the metric space $(M, d)$. Prove that $U \subset A$ is an open set in the metric space $(A, d|_A \times A)$ if and only if there exists an open set $W \subset M$ in the metric space $(M, d)$, such that

$$U = W \cap A.$$ 

Problems:
• Page 155: problems: 1, 4
• Page 156: problems: 1, 5
• Page 172: problems: 1 -just the first question.- , 5, 6, 7, 19, 30

The topics and page numbers are from the textbook.