MATH 360 — HOMEWORK 8.

due on Friday, November 2.

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Additional Reading: “Foundations of Modern Analysis”
by J. Dieudonné

Topics:
• Compact and Connected Sets
  – 3.1 Compactness
  – 3.2 The Heine-Borel Theorem
  – 3.3 Nested Set Property

Eighth Homework Assignment.

Reading:
• Read Sections 3.1, 3.2 and 3.3 paying attention to all the examples.

Exercises:

Problem 1. Prove that if $d_1$ and $d_2$ are equivalent metrics on $M$ and $T$ is a
subset of $M$, then $T$ is totally bounded in $(M, d_1)$ if and only if it is totally
bounded in $(M, d_2)$.

Problem 2. Prove that if $T$ is totally bounded in $(M, d)$, then $\overline{T}$ (the closure
of $T$) is totally bounded too.

Problem 3. Suppose $A \subset M$ is a subset of the metric space $(M, d)$. Prove
that $U \subset A$ is an open set in the metric space $(A, d|_A)$ if and only if there
exists an open set $W \subset M$ in the metric space $(M, d)$, such that

$$ U = W \cap A. $$

Problem 4. Prove that if a Cauchy sequence $\{x_n\}_{n \in \mathbb{N}}, x_n \in (M, d)$, has a
convergent subsequence, than the sequence itself is convergent.

• Page 155: problems: 1, 4
• Page 156: problems: 1, 5
• Page 172: problems: 1 -just the first question.- 5, 6, 7, 19, 30

The topics and page numbers are from the textbook.