MATH 361 — HOMEWORK 2.

due on Friday, February 1.

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Topics:
• Review of Math 360
• 5. Uniform Convergence
  – 5.1 Pointwise and Uniform Convergence
  – 5.2 The Weierstrass M Test
  – 5.3 Integration and Differentiation of Series
  – 5.5 The Space of Continuous Functions

Second Homework Assignment.

Reading:
• Read sections 5.2, 5.3 and 5.5 of Chapter 5., paying close attention
to the examples.

Exercises:

Problem 1. Let $(E, \| \|)$ be a normed vector space, $S \neq \emptyset$ a nonempty set
and $d_{\| \|}$ the metric associated to the norm $(d_{\| \|}(e, f) = \| e - f \|.)$
Prove that

$$d_{\| \|_{\infty}} = (d_{\| \|})_{\infty}.$$ 

Problem 2. Let $(E, \| \|)$ and $(F, \| \|)$ be normed vector spaces. Prove that
if $E$ is finite dimensional, then

$$L(E, F) = \mathcal{L}(E, F).$$

(That is: every linear map is continuous.)

Problem 3. Let $(E, \| \|)$ be a Banach space and $f_n : (a, b) \to E$ ($a, b \in \mathbb{R}$)
a sequence of differentiable functions such that
i) $f'_n$ converges uniformly to some function $g$.
ii) There exists $x_0 \in (a, b)$, such that $f_n(x_0)$ converges.

Prove that $f_n$ converges uniformly to some function $f$, and that $f$ is
differentiable with derivative equal to $g$. 
Remark. This is the theorem we proved in class, under the additional assumption that $f_n'$ is continuous. Instead of the Fundamental Theorem of Calculus, you will have to use the Mean Value Theorem. If you are still lost, take a look at (8.6.3) in Dieudonné.

Problems:
- Page 247: problems 3, 5
- Page 272: problems 1, 2, 3, 4, 5
- Page 316: problems 8.