MATH 361 — HOMEWORK 3.

due on Friday, February 6.

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Topics:
• Review of Math 360
• 5. Uniform Convergence
  – 5.1 Pointwise and Uniform Convergence
  – 5.2 The Weierstrass M Test
  – 5.5 The Space of Continuous Functions
  – 5.6 The Arzela-Ascoli Theorem
  – 5.7 The Contraction Mapping Principle and Its Applications

Third Homework Assignment.

Reading:
• Read sections 5.6 and 5.7 of Chapter 5., paying close attention to the
  examples. Read your notes.

Exercises:

Problem 1. Show that the initial value problem
\[
\begin{align*}
\frac{dx}{dt} &= 3x^{\frac{2}{3}} \\
x(0) &= 0
\end{align*}
\]
has both the function \( x(t) = t^3 \), and the function \( x(t) = 0 \) as solutions.

In fact \( x(t) = (t + c)^3 \) is a solution of the differential equation for every
constant \( c \).

(Note that it’s really \( (x^2)^{\frac{1}{3}} \), so it is defined for every \( x \). Obviously this
function is continuous, but not Lipschitz in any neighborhood of 0.)

Problems:
• Page 282: problems: 4, 5, 7
• Page 316: problems: 8, 18, 23, 33, 50