MATH 361 — HOMEWORK 4.

due on Friday, February 10.

by J. E. Marsden and M. J. Hoffman

Additional Reading: “Foundations of Modern Analysis”
by J. Dieudonné

Topics:
• 5 Uniform Convergence
  – 5.5 The Space of Continuous Functions
  – 5.6 The Arzela-Ascoli Theorem
  – 5.7 The Contraction Mapping Principle and Its Applications
  – 5.8 The Stone-Weierstrass Theorem

Third Homework Assignment.

Reading:
• Read Section 5.8. Read your notes.

Exercises:
Problem 1. Show that the initial value problem
\begin{align*}
\frac{dx}{dt} &= 3x^2 \\
x(0) &= 0
\end{align*}
has both the function \( x(t) = t^3 \), and the function \( x(t) = 0 \) as solutions.
In fact \( x(t) = (t + c)^3 \) is a solution of the differential equation for every constant \( c \).
(Note that it’s really \( (x^2)^{\frac{1}{3}} \), so it is defined for every \( x \). Obviously this function is continuous, but not Lipschitz in any neighborhood of 0.)

Problems:
• Page 286: problems: 2, 3, 4, 5, 6
• Page 316: problems: 26, 33, 43