MATH 361 — HOMEWORK 5.

due on Friday, October 9.

Textbook: *"Elementary Classical Analysis"* , second edition
by J. E. Marsden and M. J. Hoffman

Topics:

- **5. Uniform Convergence**
  - 5.7 The Contraction Mapping Principle and Its Applications
  - 5.8 The Stone-Weierstrass Theorem

Fifth Homework Assignment.

Reading:

- Read section 5.8 of Chapter 5., paying close attention to the examples. Read your notes.

Exercises:

Problem 1. Let $b \in \mathbb{R}$ be fixed and let $\mathcal{B} \subset C([0,1],\mathbb{R})$ be the set of all functions of the form

$$h(x) = \sum_{j=1}^{n} a_j e^{jbx},$$

where $n \in \mathbb{N}$, and $a_j \in \mathbb{R}$.
Is $\mathcal{B}$ an algebra?
Does it separate the points?
Does it vanish at any point? Does it contain the constant functions?

Problem 2. Let $\mathbb{R}_N[X]$ be the set of polynomials with real coefficients of degree at most N, and $[a,b] \subset \mathbb{R}$ a fixed interval. Prove that there exist $C, c > 0$ such that

$$c\|P\|_\infty \leq \max_{j=0}^{N}\{|a_j|\} \leq C\|P\|_\infty,$$

for every polynomial $P(X) = \sum_{j=0}^{N} a_j X^j \in \mathbb{R}_N[X]$, where $\| \|_\infty$ is the sup norm on $[a,b]$.

***Problems*:**

- Page 283: problems 6
- Page 286: problems $1, 2, 3$
- Page 316: problems $26, 41, 43$