MATH 361 — HOMEWORK 7.

due on Friday, October 30.

Textbook: *"Elementary Classical Analysis"* , second edition
by J. E. Marsden and M. J. Hoffman

Topics:

- **5. Uniform Convergence**
  - 5.7 The Contraction Mapping Principle and Its Applications
  - 5.8 The Stone-Weierstrass Theorem
  - 5.10 Power Series

Seventh Homework Assignment.

Reading:

- Read Read Section 5.10. Read your notes. Summation by parts is in the Proofs section for Section 5.9.

Exercises:

Problem 1. Prove that if the series $\sum_{n=0}^{\infty} a_n$ is convergent, then

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} k a_k = 0.$$

(Hint: Use summation by parts. This is a result of Kronecker.)

Problem 2. Prove problem 5 on page 294 by using the expansion

$$-\ln(1-z) = \frac{z}{1} + \frac{z^2}{2} + \cdots + \frac{z^n}{n} + \cdots$$

proven in class, then $\ln(1+z) - \ln(1-z)$ and finally using Abel's Theorem for $z_0 = i$.

Problem 3. Suppose $(E, \| \|)$ and $(F, \| \|)$ are normed vector spaces, and $T \in \mathscr{L}(E, F)$. Show that if for every sequence $\{x_n\}_{n\in\mathbb{N}}$, $x_n \in E$ such that $\lim_n x_n = 0$ it follows that the sequence $\{T(x_n)\}_{n\in\mathbb{N}}$ is bounded (in F), then T is continuous (i.e. $T \in L(E, F)$).

Problem 4. Suppose $(E, \| \|)$ and $(F, \| \|)$ are real normed vector spaces, and $u : E \to F$ a map satisfying

$$u(x+y) = u(x) + u(y), \quad \text{for every} \quad x, y \in E.$$

Show that if u is bounded on the unit disc $D_1(0)$, then $u \in L(E, F)$. (Hint. Show first that $u(rx) = ru(x)$, for every rational number $r \in \mathbb{Q}$.)

***Problems*:**

- Page 316: problems: $62, 63, 66$