Sylow theorems:
1. If p is a prime number such that p^k divides $|G|$, then G contains a subgroup of order p^k.
2. All Sylow p-subgroups of G are conjugate, and any p-subgroup of G is contained in a Sylow p-subgroup.
3. Let $|G|=p^k m$ with $p \nmid m$. Then the number n_p of Sylow p-subgroups satisfies $n_p|m$ and $n_p \equiv 1 \pmod p$.

Generators:
Let S be a nonempty subset of a group G. Then we have three equivalent definitions of what a subgroup $\langle S\rangle$ generated by S is:
1. $\langle S\rangle$ is the intersection of all subgroups of G which contain S.
2. $\langle S\rangle$ is the smallest subgroup containing S.
3. $\langle S\rangle$ is the set of all possible products $t_1 t_2 \cdots t_k$ of the elements of the set $S \cup S^{-1}$ where $S^{-1}=\{s^{-1} : s \in S\}$.

We say that the set S generates the group G if $G=\langle S\rangle$.

Permutations:
A k-cycle is a permutation σ of $\{1,...,n\}$ for which there are distinct indices $i_1,\dots,i_k \in \{1,...,n\}$ such that

$$\begin{aligned}
&\sigma(i_1)=i_2\\
&\sigma(i_2)=i_3\\
&\quad\vdots\\
&\sigma(i_{k-1})=i_k\\
&\sigma(i_k)=i_1
\end{aligned}$$

On indices i which are different from $i_1,\dots,i_k$ σ acts trivially $\sigma(i)=i$. We write $\sigma=(i_1\ i_2\ \dots\ i_k)$. The cycles $(i_1\ i_2\ \dots\ i_k)$ and $(j_1\ j_2\ \dots\ j_l)$ are called disjoint if the sets $\{i_1,i_2,\dots,i_k\}$ and $\{j_1,j_2,\dots,j_l\}$ are disjoint. Note that if ρ and σ are disjoint cycles, then they commute, i.e. $\rho\sigma=\sigma\rho$.

Every permutation ρ is a product of disjoint cycles. To see this first pick an arbitrary index i_1 and let $i_2=\rho(i_1)$, $i_3=\rho(i_2)$, and so on until $i_{k+1}=\rho(i_k)$ becomes i_1. Then the permutation ρ operates on the set $\{i_1,i_2,\dots,i_k\}$ the same way as the cycle $(i_1\ i_2\ \dots\ i_k)$. Now continue by picking $j_1 \notin \{i_1,i_2,\dots,i_k\}$ and defining $j_2=\rho(j_1)$, $j_3=\rho(j_2)$, etc.

A transposition is defined as a 2-cycle (ij). Note that any k-cycle can be written as a product of k-1 transpositions, namely $(i_1\ i_2\ \dots\ i_k)=(i_1 i_2)(i_2 i_3)\cdots(i_{k-1} i_k)$. This implies that any permutation can be written as a product of transpositions, i.e. that the transpositions generate the group S_n.

The representation of a given permutation ρ as a product of transpositions is not unique, even the number of transpositions that appear in the representations is not unique. However, the parity of the number of transpositions which appear in the representation of ρ is unique.

We say that ρ is even permutation if for some (and hence every) representation of ρ as a product $\tau_1\cdots\tau_k$ of transpositions τ_i we have that k is an even number. We define the sign of an even permutation to be the number +1.

We say that ρ is odd permutation if for some (and hence every) representation of ρ as a product $\tau_1\cdots\tau_k$ of transpositions τ_i we have that k is an odd number. We define the sign of an odd permutation to be the number -1.

This yields a map $\mathrm{sign}:S_n \to \{-1,1\}$ which is clearly a group homomorphism. The set of all even permutations is the kernel of this homomorphism, so it is a normal subgroup which we denote by A_n.