Cover-inclusive Dyck tilings, definition

A tiling of a skew lattice shape between 2 Dyck paths.

with Dyck tiles
(2-bin Black Dyck path)

such that: For a Dyck tile the cell entries above (i.e. has a black outline color) have the same parity as the order of a box in the usual tile, then the horizontal extent of that tile is the subset of the horizontal extent of second tile.

Example: all cover-inclusive Dyck tilings of a given shape.

For a Dyck tile $T$ of shape $d(T) = 2\lambda$, we define $\delta(T) = \delta(\lambda, \mu)$ the discrepancy between $\lambda$ and $\mu$ in number of places where $\lambda$ has a Dyck step while $\mu$ has an Up step within the Running Distance between the Dyck path of $\lambda$ and $\mu$. Let $\lambda = \{(i, j) \in \mathbb{Z}^2 \mid 0 \leq i \leq j \}$ be the number of unit squares of the skew-shape $\lambda$. We define $\varphi(T) = \bigcup T$.

THE MANY FACES AND ASPECTS OF D YCK TILINGS

The many faces and aspects of Dyck tilings

A bijection $\Phi$ between $231$-avoiding permutations in $S_n$ and Dyck tilings whose lower path is $\lambda_{n-1}$, and which contains only one-box tiles (i.e. in particular these correspond to Dyck paths, being determined just by the upper path $\lambda$).

Chord Poisets, statistics

The chords of a Dyck path $\lambda$ are the segments between matching Up and Down steps (matching parentheses in the corresponding balanced parenthesis expression). A chord with $a$ left parenthesis and $b$ right parenthesis is denoted $(a, b)$, and their lengths are written $\lambda_0$.

A chord $c$ with $a$ left parenthesis and $b$ right parenthesis is denoted $(a, b)$, and their lengths are written $\lambda_0$.

Main Results

With the definitions from Section 3, a cover-inclusive Dyck tile, definition and Chord posets, definitions.

Theorem 1: (Proposition 1 in (Kuo-Wilson 2011)) Let $\lambda$ be a Dyck path of order $\alpha$, we have

$$\sum_{\lambda \leq \alpha} \text{Chord}_{\lambda} = \left(\sum_{\lambda \leq \alpha} \text{Chord}_{\lambda}\right) - 1$$

where the sum is over all cover-inclusive Dyck tilings with fixed lower path $\lambda$.

Theorem 2: Given a Dyck path $\lambda$ of order $\alpha$, we have

$$\text{Chord}_{\lambda} \leq \sum_{\lambda \leq \alpha} \text{Chord}_{\lambda}$$

where $\lambda$ is the Jordan-Hilbert set (set of extended linear extensions) of the chord poset $\mathcal{P}$ of $\lambda$.

Proof: The two Spernerian Dyts $\mathcal{T}$ and $\mathcal{D}$, defined below, and the $\phi$-hook-length formula $e_{\mathcal{T}(\mathcal{D})} = e_{\mathcal{T}(\mathcal{D})}$ are used for Dyck tilings with lower shape $\lambda$, $\mathcal{T}$ and $\mathcal{D}$ is a bijection with $\phi(T)$.

Theorem 3: The maps $\mathcal{T}$ and $\mathcal{D}$ are bijections between interval sequences $\mathcal{T}(\mathcal{D})$ such that $\mathcal{T}(\mathcal{D}) \subseteq \mathcal{L}$ and cover-inclusive Dyck tilings of order $\alpha$.

Bijections: Linear extensions $\leftrightarrow$ Dyck tilings

The bijection: Linear extension $\mathcal{L}$ of $P$ (or $(\mathcal{D}, \mathcal{P})$) $\leftrightarrow$ Dyck tableau $\mathcal{T}$ (or $(\mathcal{D}, \mathcal{T})$).

Algorithm: Let $\lambda$ be the label of $P$ or $(\mathcal{D}, \mathcal{P})$, and let $\mathcal{T}$ be the corresponding tableau $\mathcal{T} = \mathcal{T}(\mathcal{D})$.

Proof: Let $\lambda$ be the node labeled $\lambda$, let $\mathcal{T}$ be the corresponding tableau $\mathcal{T} = \mathcal{T}(\mathcal{D})$, where $\lambda$ is the node labeled $\lambda + 1$, let $\mathcal{T}$ be the corresponding tableau $\mathcal{T} = \mathcal{T}(\mathcal{D})$, where $\lambda$ is the node labeled $\lambda + 1$, let $\mathcal{T}$ be the corresponding tableau $\mathcal{T} = \mathcal{T}(\mathcal{D})$, where $\lambda$ is the node labeled $\lambda + 1$, let $\mathcal{T}$ be the corresponding tableau $\mathcal{T} = \mathcal{T}(\mathcal{D})$, where $\lambda$ is the node labeled $\lambda + 1$, let $\mathcal{T}$ be the corresponding tableau $\mathcal{T} = \mathcal{T}(\mathcal{D})$, where $\lambda$ is the node labeled $\lambda + 1$, let $\mathcal{T}$ be the corresponding tableau $\mathcal{T} = \mathcal{T}(\mathcal{D})$, where $\lambda$ is the node labeled $\lambda + 1$, let $\mathcal{T}$ be the corresponding tableau $\mathcal{T} = \mathcal{T}(\mathcal{D})$, where $\lambda$ is 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