$$
\begin{aligned}
\sum_{k=1}^{n} \frac{2}{n}\left[\frac{(1+\frac{2k}{n})^2}{2} - (1+\frac{2k}{n}) + 1\right] &= \frac{2}{n}\sum_{k=1}^{n}\left[\frac{1+\frac{4k}{n}+\frac{4k^2}{n^2}}{2} - 1 - \frac{2k}{n} + 1\right] \\
&= \sum_{k=1}^{n} \frac{2}{n}\left[\frac{1}{2} + \frac{2k^2}{n^2}\right] \\
&= \frac{2}{n}\sum_{k=1}^{n}\left[\frac{1}{2} + \frac{2k^2}{n^2}\right] \\
&= \frac{2}{n}\left[\sum_{k=1}^{n}\frac{1}{2} + \sum_{k=1}^{n}\frac{2k^2}{n^2}\right] \\
&= \frac{2}{n}\left[\frac{n}{2} + \frac{2}{n^2}\sum_{k=1}^{n} k^2\right] \\
&= \frac{2}{n}\left[\frac{n}{2} + \frac{2}{n^2}\frac{n(n+1)(2n+1)}{6}\right] \\
&= 1 + \frac{4}{n^3}\frac{n(n+1)(2n+1)}{6} \\
&= 1 + \frac{2n(n+1)(2n+1)}{3n^3}.
\end{aligned}
$$

To find the limit we could use L'Hospital's rule, but it may be easier to use the method we used back in Section 2—divide the top and bottom by the largest power of n in the denominator:

$$
\begin{aligned}
\lim_{n\to\infty}(1 + \frac{2n(n+1)(2n+1)}{3n^3}) &= 1 + \lim_{n\to\infty}\frac{2n(n+1)(2n+1)/n^3}{3n^3/n^3} \\
&= 1 + \lim_{n\to\infty}\frac{2\frac{n}{n}\frac{(n+1)}{n}\frac{(2n+1)}{n}}{3} \\
&= 1 + \lim_{n\to\infty}\frac{2\cdot 1(1+1/n)(2+1/n)}{3} \\
&= 1 + \frac{2\cdot 1(1+0)(2+0)}{3} \\
&= 1 + \frac{4}{3} \\
&= \frac{7}{3}.
\end{aligned}
$$

So this total area under this curve on the interval $[1,4]$ is exactly $7/3$.