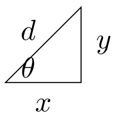
4

a and b



so $y = d\sin\theta$ and $x = d\cos\theta$.

c

We use implicit differentiation: $\frac{dy}{dt} = \frac{dd}{dt}\sin\theta$ and $\frac{dx}{dt} = \frac{dd}{dt}\cos\theta$.

d

$y(t) = -\frac{g}{2}t^2 + v_y t = -\frac{g}{2}t^2 + (v\sin\theta)t$. (We can plug in to see that $y(0) = 0$, and take the derivatives to check the other two requirements. Solving $y(t) = 0$, we get $t = 0$ or $t = \frac{2v\sin\theta}{g}$. This second one must be when the cannonball returns to the ground.

e

The velocity is $v\cos\theta$, so it hits the ground after traveling $\frac{2v^2\sin\theta\cos\theta}{g} = \frac{v^2\sin 2\theta}{g}$.

f

If the angle is $\pi/2$, we're sending the ball straight up, so it is shouldn't be travelling horizontally at all. Indeed, $\cos\pi/2 = 0$, so the horizontal distance travelled in this case is 0.

g

We want $\frac{50^2}{g} = \frac{v^2\sin 2(\pi/4)}{g} = \frac{v^2}{g}$, so $v = 50$.

h

We know that the distance the ball travels, as a function of v and θ, is $x = \frac{v^2\sin 2\theta}{g}$.

Let's suppose we make an error in v: θ is constant, and we view x as a fuction of v. Then $dx = 2\frac{v\sin 2\theta}{g}dv$, and $\frac{dx}{x} = \frac{2\frac{v\sin 2\theta}{g}dv}{\frac{v^2\sin 2\theta}{g}} = 2\frac{dv}{v}$. When the percentage error is 5%, so $\frac{dv}{v} = 0.05$, this is an error of 10%.

Instead, let's suppose we make an error in θ: v is constant, and we view x as a function of θ. Then $dx = \frac{v^2}{g} 2\cos 2\theta d\theta$ and $\frac{dx}{x} = \frac{\frac{v^2}{g} 2\cos 2\theta d\theta}{\frac{v^2 \sin 2\theta}{g}} = 2\cot 2\theta d\theta$.