

Math 103, Fall 2014
Week 10

In Class Work, Tuesday, October 28th

Exercise 1

Consider the polynomial $x^3 + 2x - 4$.

- (a) Show that this polynomial has at least one root.

Remember, a root is a value a so that $a^3 + 2a - 4 = 0$. You can't show this directly, so you'll have to give an argument. This problem doesn't need any of the new material; we've done problems like it before.

- (b) Show that this polynomial has at most one root.

This is what we need the new material for.

- (i) Suppose not--suppose that there are two distinct roots, a and b . You may as well assume $a < b$ (otherwise we'd just swap the names).
- (ii) What do you know about the value of the function $f(x) = x^3 + 2x - 4$ at a and b ?
- (iii) What does Rolle's Theorem tell you about this situation? Make sure to check the assumptions of Rolle's Theorem explicitly.
- (iv) If you got something impossible, your supposition--that there were two distinct roots--must have been wrong, so there must be only one root.
- (v) When you write up your solution, write it as a list of steps, and each step should be a single fact, like introducing a name, making a calculation, or using previous steps to derive a new fact.

Put up a green flag when you're done.

Exercise 2

Consider the function $f(x) = e^x - 4$.

- (a) Show that the equation $f(x) = 0$ has at least one solution.

We aren't calling them "roots" anymore because only polynomials have roots.

- (b) Show that the equation $f(x) = 0$ has at most one solution.

Again, your final solution should be a clearly written argument with a list of easy-to-follow steps.

Exercise 3

Draw a graph of a function $f(x)$ which is continuous at every point of the closed interval $[-2, 2]$ so that $f(-2) = 0 = f(2)$ but the derivative of f is not 0 anywhere in $(-2, 2)$.

Exercise 4

A person drives the 300 miles from Philadelphia to Pittsburgh in 6 hours. (The car starts and ends parked, at 0 mph.)

- (a) What was this person's average speed?
- (b) How do you know this person was driving at a speed of exactly 50 miles per hour at least once during the trip?
- (c) How do you know this person was driving at a speed of exactly 40 miles per hour at least twice during the trip?

Exercise 5

Consider the polynomial $x^4 + 4x^2 - 10x + 1$.

- (a) Show that this polynomial has at least two roots.
- (b) Show that this polynomial has at most two roots.