Math 103, Fall 2014
Week 10

In Class Work, Thursday, October 30th

Warm Up

Identify the inflection points of $x^2(x-2)$.

Exercise 1

1. Does x^4 have a critical point at 0? Why or why not?

2. Does x^4 have an inflection point at 0? Why or why not?

Exercise 2

Sketch a graph of the function $f(x) = \frac{x^2+x-2}{x+3}$.
   There's a lot of information to organize here.

 (i) Where is $f(x)$ undefined?

 (ii) For which values of x is $f(x) = 0$?

(iii) What intervals is $f(x)$ positive on?

 (iv) Identify horizontal and vertical asymptotes.

 (v) Find $f'(x)$.

 (vi) Find the critical points of $f(x)$.

(vii) What intervals is $f(x)$ increasing or decreasing on?

(viii) Find $f''(x)$.

 (ix) Find the critical points of $f'(x)$.

(x) What intervals is $f(x)$ concave up or concave down on?

(xi) You may find it helpful to organize all this information into
     a table (or not--figure out what works for you)

(xii) Sketch the graph!

Exercise 3

(a) Sketch a graph of a function which is defined on the interval $[0, 2]$, is
concave down on the interval $[0, 1]$, and concave up on the interval
$[1, 2]$.

(b) Sketch a graph of a function $u(x)$ which is defined everywhere, so that
$u'(x)$ is defined and positive for all x, and such that $\lim_{x \to -\infty} u(x) = -2$
and $\lim_{x \to \infty} u(x) = 2$.

Exercise 4

Sketch the graph of $f(x) = \frac{x^2 - 1}{x^2 + 1}$.

Exercise 5

(a) a and b are unknown constants and $g(x) = xe^{ax} + bx$ is an unknown
function. You know that $g(x)$ has an absolute minimum at the point
$(1, -e)$; what does this tell you about $g'(1)$?

(b) Use what you know about $g'(1)$ to find an equation relating a and b.

(c) Use the fact that $g(1) = -e$ to solve for a and b.