Math 103, Fall 2014
Week 11

After Class Homework
Due Monday, November 17

1. Find $\lim_{x\to\infty} x^4 e^{-x}$.
2. Find $\lim_{x\to 0^+} x(\ln x)^2$.
3. Find $\lim_{x\to 0^+}(\ln x - \ln\sin x)$. (Hint: use a logarithm rule to turn this into a fraction, then use the continuity of ln to pull ln outside the limit.)
4. Find $\lim_{x\to\infty} x^{\frac{1}{\ln x+1}}$.
5. Find $\lim_{x\to 0^+}(1+\frac{1}{x})^x$.
6. We go see a movie in a theater where the screen is 16 feet tall, with the bottom elevated 9 feet off the ground. The theater is 100 ft deep. How far back should we sit to get the best seat (specifically, to maximize the fraction of our visual field taken up by the screen)?
7. A farmer wants to construct a rectangular pen next to a barn which is 60 feet long. *Part* of one side of the pen will be the barn, but that side (might) extend beyond the barn. Find the dimensions of the pen with the largest area that the farmer can build if 300 feet of fencing material is available.
8. A math professor is playing fetch with his Corgi dog at the shore. (The shore is a straight line separating the beach from the water.) He stands on the edge of the water and throws the ball 5 meters straight into the water, perpindicular to the shoreline. The dog is also standing on the shore, 15 m away from the professor. The dog runs part way along the shoreline, then jumps in the water and swims to the ball. If the dog can run at a speed of 2 m/s and swim at a speed of 1 m/s, and the dog wants to get to the ball in the shortest possible time, where does the dog jump in the water?
9. We make a steel grain silo with total volume 300π cubic feet. The silo consists of a steel cylinder with a hemisphere roof *but no floor*. What dimension should the silo have to minimize the amount of material used? (This is very similar to the problem in the video, except that this time there's no floor.)