Math 103, Fall 2014
Week 11

In Class Work, Tuesday, November 4th

Warm Up

Find $\lim_{x \to 0} \frac{\cos x - 1}{x^2}$.

Exercise 1

(a) Give examples of functions $u(x)$ and $v(x)$ so that:

- $\lim_{x \to 2} u(x) = 0$,
- $\lim_{x \to 2} v(x) = 0$, and
- $\lim_{x \to 2} \frac{u(x)}{v(x)} = 0$.

(b) Give examples of functions $s(x)$ and $t(x)$ so that:

- $\lim_{x \to 2} s(x) = 0$,
- $\lim_{x \to 2} t(x) = 0$, and
- $\lim_{x \to 2} \frac{s(x)}{t(x)} = \infty$.

Exercise 2

(a) What kind of indeterminate form is $\lim_{x \to 0^+} x^2 \ln x$?

(b) Combine algebra with L'Hospital's rule to find $\lim_{x \to 0^+} x^2 \ln x$.

`(i) L'Hospital's rule only works on fractions, so use the fact that` $x^2 = \frac{1}{\frac{1}{x^2}}$ `to rewrite this as a fraction.`

`(ii) Check that you have an indeterminate form and apply L'Hospital's rule.`

Exercise 3

(a) Find $\lim_{x\to\pi}(\csc x)(\ln \frac{x}{\pi})$.

(b) In the first part you had a choice: you could have written it as $\lim\limits_{x\to 0} \frac{\ln \frac{x}{\pi}}{\frac{1}{\csc x}}$ or $\lim\limits_{x\to 0} \frac{\csc x}{\frac{1}{\ln \frac{x}{\pi}}}$. Which did you choose, and why?

Exercise 4

Suppose $f(x)$ and $g(x)$ are unknown functions, and we want to know what $\lim_{x\to 4} \frac{f(x)}{g(x)}$ is.

(a) Suppose you know that $\lim_{x\to 4} f(x) = 3$ and $\lim_{x\to 4} g(x) = 7$. Is this enough information to figure out what $\lim_{x\to 4} \frac{f(x)}{g(x)}$ is? If so, what is it (and how do you know)? If not, give a convincing argument that this is not enough information.

(b) Suppose you know that $\lim_{x\to 4} f(x) = 0$ and $\lim_{x\to 4} g(x) = 6$. Is this enough information to figure out what $\lim_{x\to 4} \frac{f(x)}{g(x)}$ is? If so, what is it (and how do you know)? If not, give a convincing argument that this is not enough information.

(c) Suppose you know that $\lim_{x\to 4} f(x) = 2$ and $\lim_{x\to 4} g(x) = 0$. Is this enough information to figure out what $\lim_{x\to 4} \frac{f(x)}{g(x)}$ is? If so, what is it (and how do you know)? If not, give a convincing argument that this is not enough information.

(d) Suppose you know that $\lim_{x\to 4} f(x) = 0$ and $\lim_{x\to 4} g(x) = 0$. Is this enough information to figure out what $\lim_{x\to 4} \frac{f(x)}{g(x)}$ is? If so, what is it (and how do you know)? If not, give a convincing argument that this is not enough information.

Exercise 5

(a) What kind of indeterminate form is $\lim_{x\to 0^+}(\csc x - \cot x)$?

(b) Combine algebra with L'Hospital's rule to find $\lim_{x\to 0^+}(\csc x - \cot x)$.

(i) Write these trig functions as fractions (for instance, $\csc x = \frac{1}{\sin x}$)

(ii) Put them over a common denominator.

(iii) Check that you have an indeterminate form and L'Hospital's rule.

Exercise 6

Find $\lim_{x\to 1^+}(\frac{1}{x-1} - \frac{1}{\ln x})$.
You'll want to find a different way to turn this into a fraction.

Exercise 7

(a) What kind of indeterminate form is $\lim_{x\to 0^+} x^x$?

(b) Combine algebra with L'Hospital's rule to find $\lim_{x\to 0^+} x^x$.

(i) Use logarithm rules to rewrite $\lim_{x\to 0^+} x^x = e^{\ln \lim_{x\to 0^+} x^x}$

(ii) Use continuity to bring $\ln$ inside the limit

(iii) Use the logarithm rule to rewrite $\ln x^x = x \ln x$

(iv) Now you have an indeterminate product. Use the techniques you learned above to find this limit.

Exercise 8

(a) What kind of indeterminate form is $\lim_{x\to 0}(\cos x)^{1/x}$?

(b) Find $\lim_{x\to 0}(\cos x)^{1/x}$.

Exercise 9

(a) Explain why 0^∞ is *not* an indeterminate form.

(b) Find $\lim_{x\to\infty}(1/x)^x$.

Exercise 10

(a) Explain why L'Hospital's rule doesn't help you find $\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}$.

(b) Find $\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}$ another way.