Math 103, Fall 2014
Week 11

In Class Work, Thursday, November 6th

Exercise 1

Set up, but do not finish solving, this problem.

We go see a movie in a theater where the screen is 16 feet tall, with the bottom elevated 9 feet off the ground. The theater is 100 ft deep. How far back should we sit to get the best seat (specifically, to maximize the fraction of our visual field taken up by the screen)?

(i) Draw a picture first.

(ii) There is a quantity we choose. Make sure this quantity is called x in your picture.

(iii) What are appropriate bounds on x?

(iv) There is a quantity we are trying to optimize. Make sure this quantity is called θ in your picture.

(v) Write an equation relating θ to x. (This step is tricky, and may require you to identify other interesting numbers in the picture.)

(vi) Stop here: you'll finish this at home.

Exercise 2

Set up, but do not finish solving, this problem.

A farmer wants to construct a rectangular pen next to a barn which is 60 feet long. *Part* of one side of the pen will be the barn, but that side (might) extend beyond the barn. (That side *must* be at least 60 feet long, since it includes the barn.) Find the dimensions of the pen with the largest area that the farmer can build if 300 feet of fencing material is available.

(i) `Draw a picture.`

(ii) `You should end up with three variables: one for each dimension of the fence, and one for the area.`

(iii) `Write a` ***`constraint`*** `equation relating the dimensions of the fence to each other.`

(iv) `Write an equation relating the area to the dimensions of the fence.`

(v) `Stop here: you'll finish this at home.`

Exercise 3

We'll actually solve this one all the way through.

You have just been appointed as the top energy advisor to the king of Gondor.* The king wishes to devise a policy package that will reduce emissions from the industrial sector. He is willing to impose up to $10 million of net costs on the Industrial Sector. The king is considering three policies. Each policy becomes more expensive as it becomes more stringent (as the most cost-effective opportunities to satisfy each policy are exhausted and industries are forced to implement more costly measures). (CO_2e means "CO_2-equivalent", and is a measurement of overall reduction in greenhouse gasses. Reducing a ton of CO_2 means one ton of CO_2e reduction, but reducing a ton of another gas can be more or less than one ton of CO_2e, depending on the gas.)

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*This question is a bit different because it was originally written as part of a pre-hiring assessment at an environmental consulting company. (It wasn't written by anyone affiliated with the course; the author was kind enough to share it with us.) It's less streamlined than most of the questions we've done, but 1) by this point in the course, you're ready for that, and 2) that was part of the point of the original question, and I didn't want to take that away.

1. Industries may capture or flare methane currently leaking to the atmosphere. The cost to abate x tons of CO_2e in this way is $\$(-30x+\frac{1}{4000}x^2)$.
2. Industries may substitute other chemicals for SF_6. The cost to abate x tons of CO_2e in this way is $\$(10x+\frac{1}{20000}x^2)$. †

How many tons should industry be required to abate under each of the king's policies in order to achieve the maximum total amount of abatement without exceeding the king's \$10 million net cost limit? (Gondor's industry is sufficiently large that industry size is not a constraint on the amount of emissions that may be abated under any of these policies.)

Exercise 4

Set up, but do not finish solving, this problem.

A math professor is playing fetch with his Corgi dog at the shore. (The shore is a straight line separating the beach from the water.) He stands on the edge of the water and throws the ball 5 meters straight into the water, perpindicular to the shoreline. The dog is also standing on the shore, 15 m away from the professor. The dog runs part way down the beach then jumps in the water and swims to the ball. If the dog can run at a speed of 6 m/s and swim at a speed of 1 m/s, and the dog wants to get to the ball the fastest he can, where does he jump in the water?‡

We'll call the time it takes the dog to get to the ball t and we'll call the distance from where the dog jumps in to where the man is standing x.

(i) `Make sure to draw a picture.`

(ii) `Identify appropriate bounds on` x.

(iii) `Express the time it takes the dog to reach the ball as a function of` x.

(iv) `Stop here: you'll finish this at home.`

† From the author: "Methane and especially SF_6 are powerful greenhouse gases. This background information does not affect one's ability to solve the math problem, but it explains why these two policies are able to achieve CO_2 abatement, despite the fact that they deal with non-CO_2 gases."

‡ This problem is adapted from an article by Professor Tim Pennings in the College Mathematics Journal 34(May):178-182, 2003. He had a hunch that his dog was minimizing the time it took to reach the ball and so he gathered empirical evidence to support his hunch.

Exercise 5

Some follow-up on the question about Gondor.

(a) What is the *marginal cost* of abating one more ton of CO_2 using the first policy when x tons have already been abated with that policy?

(b) What is the *marginal cost* of abating one more ton of CO_2 using the second policy when x tons have already been abated with that policy?

(c) At the optimal solution you found, what are the marginal costs of each of the two policies?

(d) There should be something interesting about your answer to the previous part. What is it, and why does that have to be the case?

(e) The original problem actually had *three* policies, and was stated in terms of marginal cost:

  (a) Industries may capture or flare methane currently leaking to the atmosphere. The marginal cost per ton CO_2e abated is: negative \$30 plus (0.0005 times the number of tons abated).

  (b) Industries may substitute other chemicals for SF_6. The marginal cost per ton CO_2e abated is: positive \$10 plus (0.0001 times the number of tons abated).

  (c) Industries may improve the energy efficiency of their equipment. Given the current mix of energy sources used by Gondor's industry, the marginal cost per ton CO_2e abated is: negative \$20 plus (0.001 times the number of tons abated).

  Note that the total cost of abating x tons of CO_2e using the third policy is $\$(-20x + \frac{1}{2000}x^2)$.

  If all three policies are available, how many tons should industry be be required to abate under each of the king's policies in order to achieve the maximum total amount of abatement without exceeding the king's \$10 million net cost limit? (You'll need to use your observation about the relationships between the marginal costs at the optimum.)