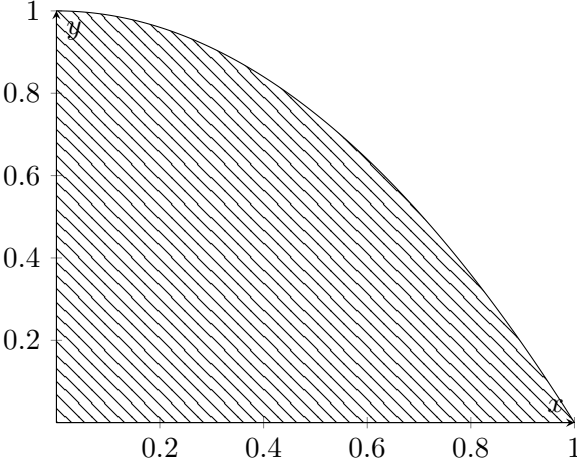# Math 103, Fall 2014
Week 12

In Class Work, Tuesday, November 11th
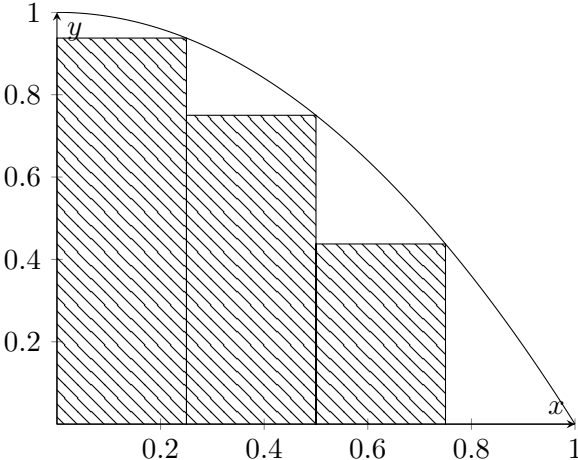
Warm Up

1. (a) Rewrite the sum $\sum_{k=0}^{4} \cos \frac{k\pi}{2}$ without sigma notation.

   (b) Rewrite $1 + 3 + 5 + 7 + 9 + 11$ with sigma notation.

2. Use the algebra rules for finite sums to evaluate these sums:

   (a) $\sum_{k=1}^{4} 2k^2$

   (b) $\sum_{k=1}^{8} 4k + 3k^2$
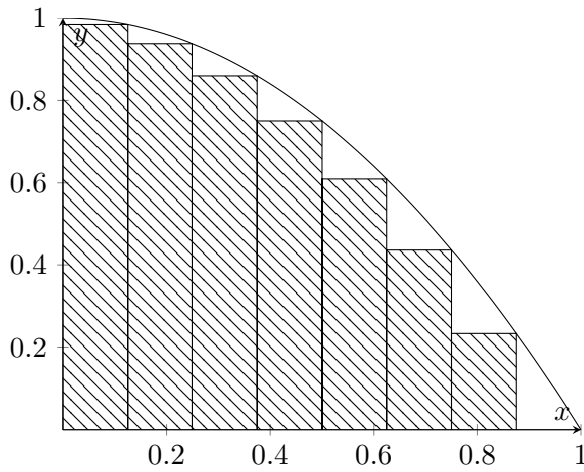
   (c) $\sum_{k=1}^{n} 5k^3 - 2k$

Exercise 2

We're going to try to estimate the area under the parabola $y = 1 - x^2$ between 0 and 1:

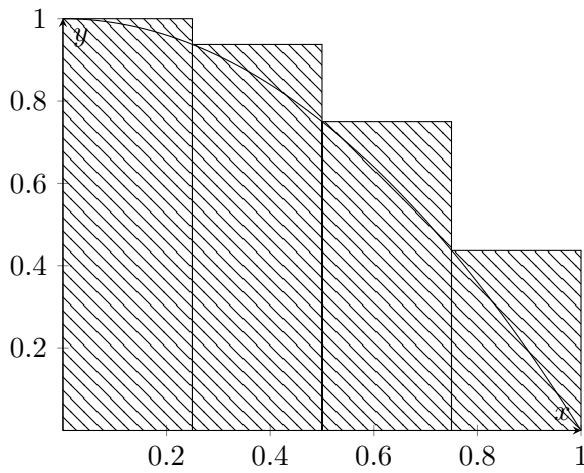(a) Calculate the shaded area in the picture below:

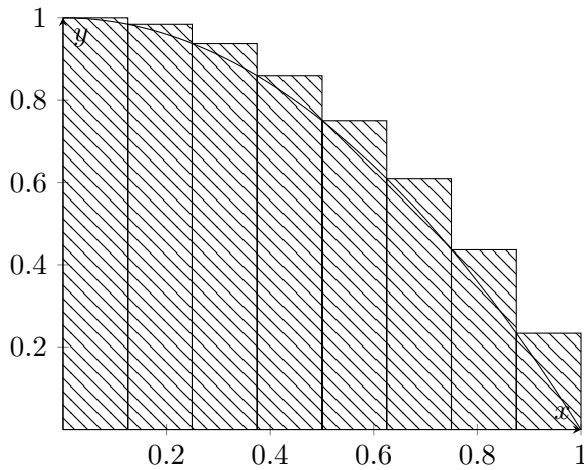(b) Calculate the shaded area in the picture below:

(c) Are the estimates from the previous two parts larger or smaller than the actual value?

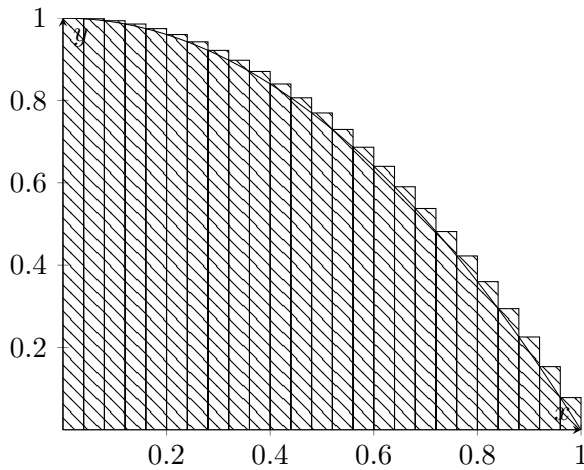(d) Calculate the shaded area in the picture below:

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(e) Calculate the shaded area in the picture below:

(f) Are the estimates from the previous two parts larger or smaller than the actual value?

To make the estimate better, we could use even more rectangles:

There are 25 rectangles in this picture.

(g) What is the area of the 7th rectangle?

(h) What is the area of the 12th rectangle?

(i) If k is an integer between 1 and 25, what is the area of the kth rectangle?

Once again, we can use the left endpoint instead of the right endpoint.

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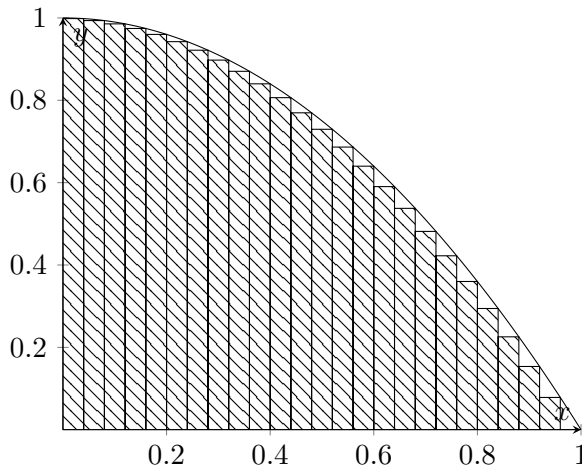
There are 25 rectangles in this picture (if you count the last one, which has height 0).

(j) What is the area of the 9th rectangle?

(k) What is the area of the 15th rectangle?

(l) If k is an integer between 1 and 25, what is the area of the kth rectangle?

(m) Write the area in these rectangles using Σ notation.

(n) Use the algebra rules for Σ notation to calculate the are in these 25 rectangles.

(o) Based on this estimate, estimate the average value of the function $1 - x^2$ (using the definition of the average value of a function).

Exercise 3

(a) According to the definition of the average value of a function, what is the average value of $f(x) = 2$ on the interval $[-1, 1]$?

(b) Does that answer make sense? Why or why not?

(c) According to the definition of the average value of a function, what is the average value of $f(x) = x + 1$ on the interval $[-1, 1]$?

(d) Does that answer make sense? Why or why not?

Exercise 4

Consider the sum $\sum_{k=1}^{100}(A + kd)$.

    (a) If you expanded this to a sum without sigma notation, which variables would still appear in that expression?

    (b) Use the algebra rules for finite sums to evaluate this sum.

Exercise 5

There are many rules more complicated than the left and right endpoints we could use—for instance, we could use the midpoint.
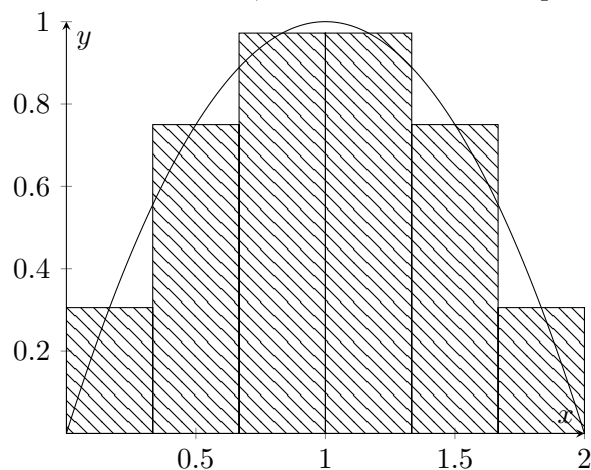
    (a) Express the shaded area using Σ notation.

    (b) Calculate the shaded area any way you want.

Exercise 6

    (a) Draw a function where the left endpoint rule *overestimates* the area.

    (b) Draw a function where the left endpoint rule *underestimates* the area.

    (c) Draw a function where the right endpoint rule *overestimates* the area.

    (d) Draw a function where the right endpoint rule *underestimates* the area.

    (e) Do these examples have anything to do with the second derivative of the function?