Math 103, Fall 2014
Week 12

In Class Work, Thursday, November 13th

Warm Up

1. Suppose g and h are integrable functions such that $\int_0^3 g(x)dx = 2$, $\int_2^3 h(x)dx = 1$, and $\int_0^2 g(x) = -1$.

   (a) What is $\int_2^3 g(x)dx$?

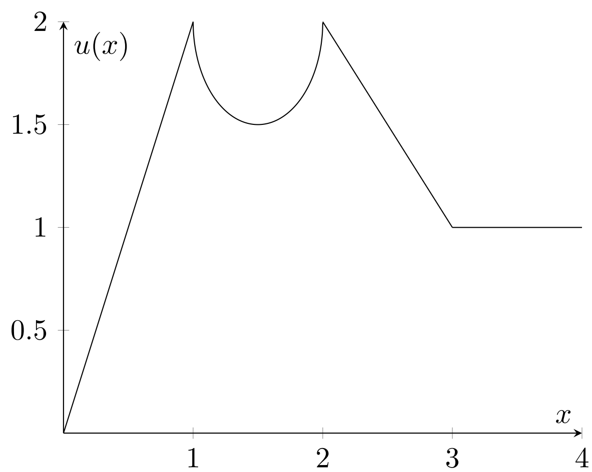
   (b) What is $\int_2^3 g(x) + h(x)dx$?

   (c) What is $\int_2^2 g(x)dx$?

   (d) What is $\int_2^3 2g(x) - 4h(x)dx$?

   (e) What is the average value of $g(x)$ on the interval $[0, 3]$?

2. Consider the following strange function:



What is $\int_0^4 u(x)dx$?

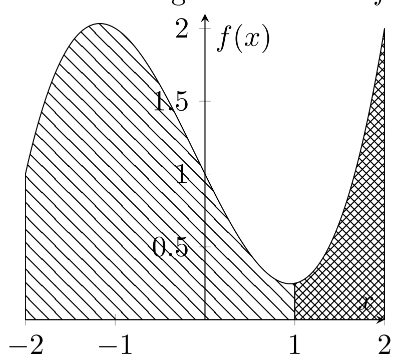
Exercise 1

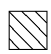
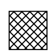
(a) Explain why L'Hospital's rule doesn't help you find $\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}$.

(b) Find $\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}$ another way.

Exercise 2

Consider the integrable function $f(x)$ below:



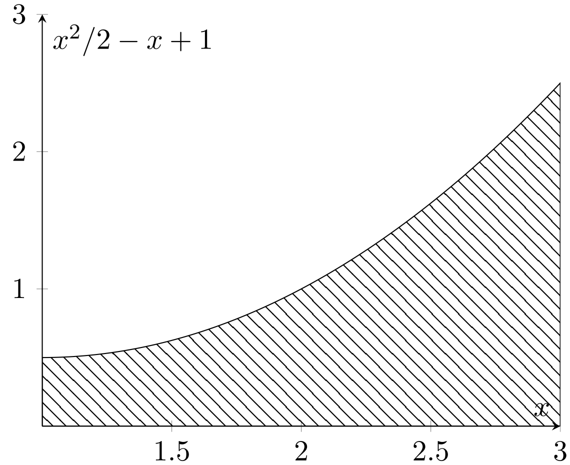
1. Express the area shaded like  as a definite integral.
2. Express the area shaded like  as a definite integral.
3. Express the combined shaded area as a definite integral.
4. Write an equation relating these three integrals.
5. Which rule from Table 5.4 on page 317 does this represent?

Exercise 3

Without doing any calculations (but feel free to draw a picture), which is larger, $\int_1^2 x^2 dx$ or $\int_1^2 x\, dx$?

Exercise 4

We're going to put together everything we've seen to calculate $\int_1^3 x^2/2 - x + 1\,dx$.



(a) As a warm up, estimate the area under this curve using four rectangles

(b) Now calculate $\int_1^3 x^2/2 - x + 1\,dx$.

  (i) `Suppose we divide the area under the curve into` n `rectangles, where` n `is some big number. What is the width of each rectangle?`

  (ii) `When there are` n `rectangles, what is the height of the 1st rectangle? What about the 5th? The` k`th?`

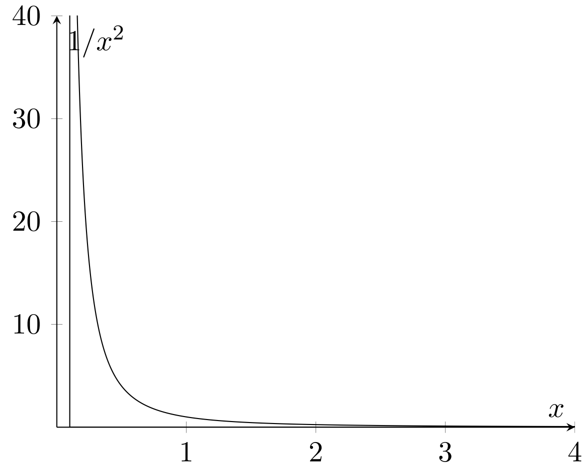
  (iii) `Express the area under` n `rectangles using` Σ `notation.`

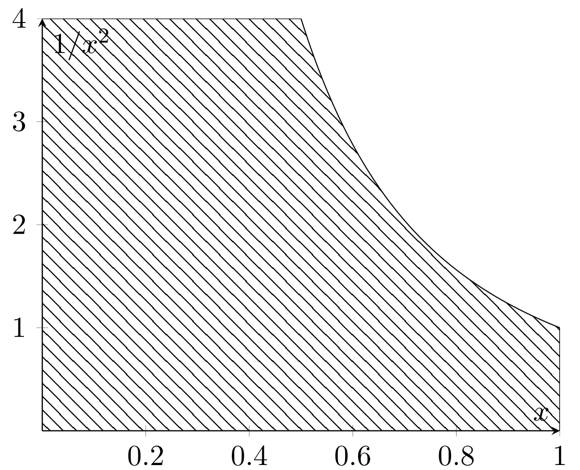
  (iv) `Use the rules for` Σ `notation to calculate this sum.`

  (v) `Take the limit as` $n \to \infty$`. The value of this limit is the definite intgral.`

Exercise 5

We want to check that $\int_0^1 x^{-2}dx$ is actually infinite. We will do this by showing that $\int_0^1 x^{-2}dx \geq N$ for all N.





Both graphs are showing $1/x^2$; the first one gives a bigger picture, and the second one is zoomed in to only show that part where x is between 0 and 1 and y is between 0 and 4. Don't get confused by the fact that the picture is cut off: as x gets closer to 0, $1/x^2$ gets big, so the region underneath the curve $1/x^2$ includes things above $y = 4$.

(a) Consider the box underneath the curve $1/x^2$ which has width 1 and is as tall as possible. Draw this box in the second picture above and calculate its area.

(b) Consider the box underneath the curve $1/x^2$ which has width $1/2$ and is as tall as possible. Draw this box in the picture above and calculate its area.

(c) What is the area of the largest possible box which fits underneath the curve $1/x^2$ and has width $1/4$?

(d) Find a box contained underneath the curve $1/x^2$ with area $\geq N$ for an arbitrary N?

(e) Explain why this means $\int_0^1 x^{-2} dx$ must be infinite.

In 104 you'll see that some curves can have finite area even though they approach infinity. For instance, you'll see that $\int_0^1 x^{-1/2} dx = 2$.

Exercise 6

We can view the definite integral as a function itself.

1. Define a function $F(t) = \int_0^t x \ dx$? Use geometry to express $F(t)$ as a polynomial.

   `Draw a picture of the area represented by the integral` $\int_0^t x \ dx$.

2. Define a function $G(t) = \int_2^t x \ dx$? Use geometry to express $G(t)$ as a polynomial.