Math 103, Fall 2014
Week 13

In Class Work, Thursday, November 20th

Warm Up

1. (a) Write down a particular antiderivative of $3x^4$. (It should *not* have an arbitrary constant in it.)
   (b) Write down a different particular antiderivative of $3x^4$.
2. (a) Write down an antiderivative of e^{2x}.
   (b) Write down $\int e^{2x}dx$ (the *indefinite integral*) of e^{2x}.
   (c) Write down a particular antiderivative of e^{2x}, $F(x)$, with the property that $F(1) = 2$.

Exercise 1

(a) Write down a particular antiderivative of $3x^4 + 4x^3 - \cos x$.

`Make sure to take the derivative of your answer to check that you're correct.`

(b) Write down a particular antiderivative of $4e^{2x} + \frac{2}{1+x^2} - 3\sec x + 2^x$.

Exercise 2

Your friend proposes a product rule for antiderivatives:

> If $F(x)$ is the antiderivative of $f(x)$ and $G(x)$ is the antiderivative of $g(x)$ then $F(x)G(x)$ should be the antiderivative of $f(x)g(x)$.

Demonstrate that this isn't always true.

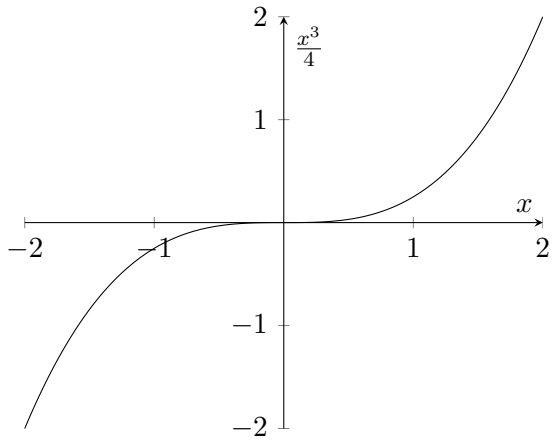
Exercise 3

(a) Find the indefinite integral $\int \frac{4}{x^2} dx$.

(b) Use Part 2 of the Fundamental Theorem of Calculus to find the definite integral $\int_1^2 \frac{4}{x^2} dx$.

(c) Explain why Part 2 of the Fundamental Theorem of Calculus doesn't tell you how to find $\int_{-1}^2 \frac{4}{x^2} dx$. (You'll learn how to solve this integral next semester.)

Exercise 4



(a) Find $\int_{-2}^2 |\frac{x^3}{4}| dx$. (Careful: you don't know any antiderivative for $|\frac{x^3}{4}|$!)

(b) Find $\int_{-\pi}^{\pi/3} |\sin x| dx$.

Exercise 5

If $F'(t)$ is the *rate of change* at time t then $\int_a^b F'(t)dt$ is the *net change*—the overall amount of change—between times a and time b.

(a) We throw a rock straight up into the air at time 0. At time t, the velocity of the rock is $v(t)$. (We don't know an exact formula for $v(t)$.) Write down an equation involving v, the time t, and an integral which will be true when the rock returns to its starting height (and at no other time except perhaps time 0).

(b) From the information given, is it possible to write down an equation that will be true when the rock hits the ground?

(c) What is the difference between these two examples? What does it have to do with the constant of integration?

Exercise 6

(a) If $F(x) = \int_0^x t^2 dt$, find a polynomial which is equal to $F(x)$.

(b) If $G(z) = F(z^2)$, find $G(z)$ as a polynomial.

(c) Find $G'(x)$.