Math 103, Fall 2014
Week 14

In Class Work, Tuesday, November 25th

Warm Up

(a) What is $\frac{d}{dx}\int_1^x 3\cosh(e^t)dt$?

(b) What is $\frac{d}{dx}\int_x^1 7\frac{\sin t}{t}dt$?

Exercise 1

(a) Consider the function $F(x) = \int_1^x \ln t\,dt$. Use Part 1 of the Fundamental Theorem of Calculus (page 327) to find $\frac{d}{dx}F(x)$.

(b) Now consider the function $G(x) = \int_1^{x^3} \ln t\,dt$. $G(x) = F(?)$; what is "?"?

(c) What is $\frac{d}{dx}G(x)$?

(d) What is $\frac{d}{dx}\int_1^{\ln x} \frac{\sin t}{t}dt$?

(e) What is $\frac{d}{dx}\int_{1/x}^{\ln x} \frac{\sin t}{t}dt$?

Exercise 2

(a) Find $\frac{d}{dz}\int_0^{z^2} t^2 dx$ using the chain rule and FTC.

(b) How does this compare to the last question from last week? (Do that question if you haven't already.)

Exercise 3

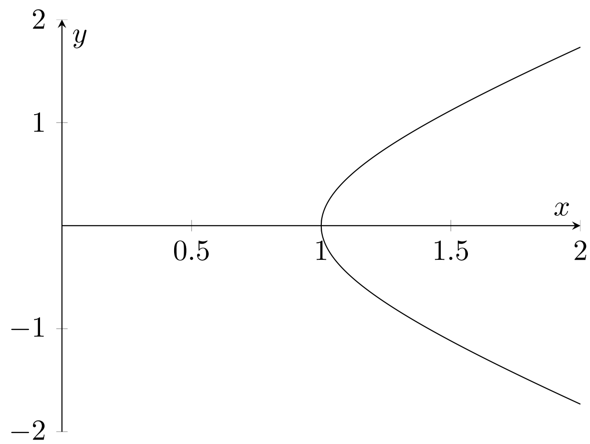
The marginal revenue for a platypus farm to raise and sell a pet platypus is $\frac{dr}{dx} = 2 + 4x - x^2/2$ where r is revenue measured in hundreds of dollars and x is measured in hundreds of platypodes. (That is, if x hundred platypodes have already been sold, the next platypus will bring in $2 + 4x - x^2/2$ in additional revenue. Go ahead and assume that platypodes are continuous, so that it's okay to buy and sell fractional platypodes.)

(a) What will the total revenue be for raising and selling 2 hundred platypodes?

(b) How many platypodes should the farm raise to maximize their revenue?

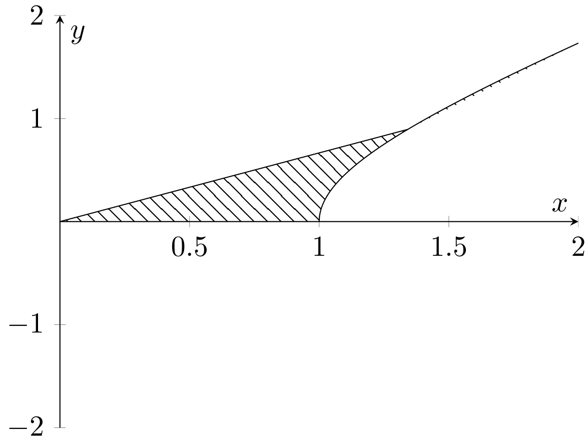
Exercise 4

The hyperbolic trig functions are named after their relationship to the hyperbola $x^2 - y^2 = 1$:

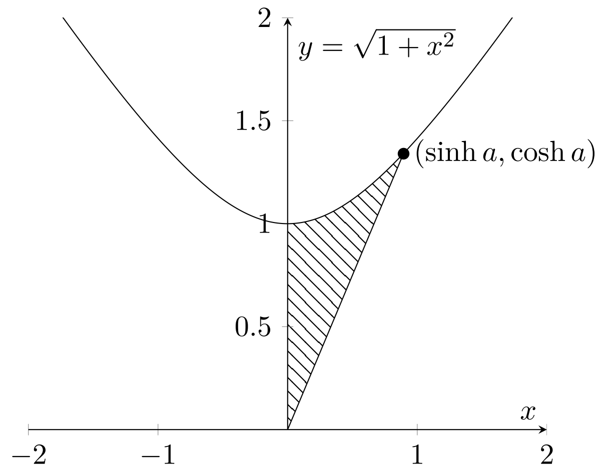


The points on this curve have the form $(\cosh a, \sinh a)$ for various values of a, analogous to the way the points on the circle are $(\cos\theta, \sin\theta)$. a is sometimes called the "hyperbolic angle".

We will identify the area cut out by the hyperbolic angle:



It will be easier if we rotate the graph:



Here a is some arbitrary (positive) number, and we want to find the area as a function of a.

(a) Write an equation for the line which forms the lower boundary of the region.

(b) Express the shaded region as a difference of two definite integrals.

(c) Next semester you'll learn a technique for integrating $\sqrt{1+x^2}$. In the meantime, check that

$$\int \sqrt{1+x^2}dx = \frac{1}{2}(x\sqrt{1+x^2} + \sinh^{-1} x) + C.$$

(d) Use this to find the shaded area (as a function of a).

(e) Compare to the same calculation for a circle: