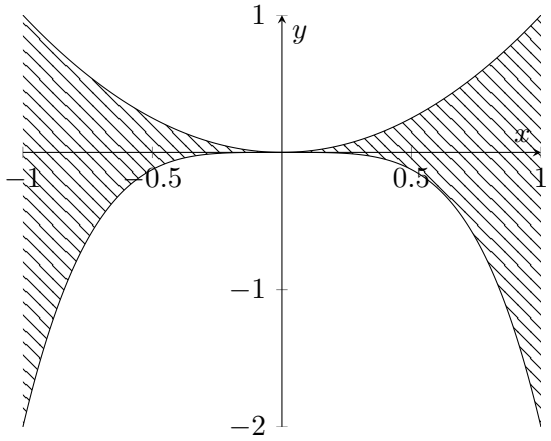
Math 103, Fall 2014
Week 15

In Class Work, Thursday, December 4th

Note: the answers I gave to compare to are mostly decimals, but that's not the most useful way to solve the problem—you should leave your answers as fractions or sums and differences of fractions, and just use the information provided to check that you're correct.

Exercise 1

Consider the area enclosed by the curves $y = x^2$ and $y = -2x^4$ and the lines $x = -1$ and $x = 1$:

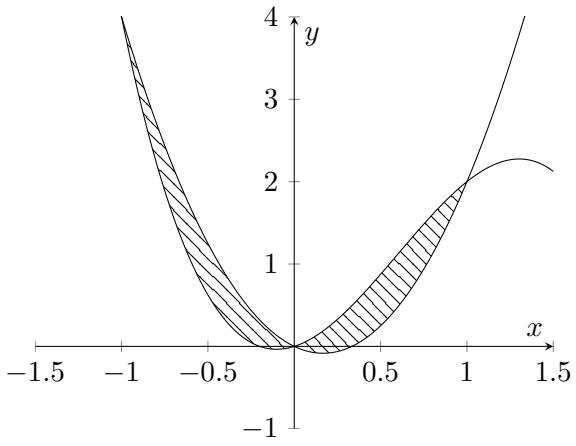


(a) Express this area as a single integral.

(b) Find the area.

Exercise 2

The curves $y = -\frac{5}{3}x^3 + 3x^2 + \frac{2}{3}x$ and $y = 3x^2 - x$ interesect three times, at $(-1, 4)$, at $(0, 0)$, and at $(1, 2)$. Find the area enclosed by these two curves.



Exercise 3

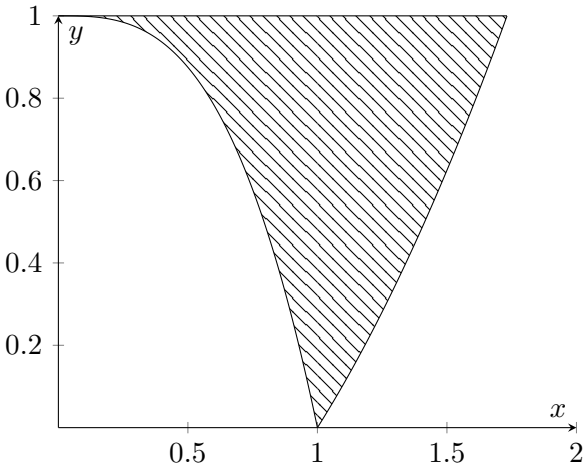
The curves $y = 8 - x^6$ and $y = 7x^3$ intersect in two places, enclosing an area.

(a) Find the two points where these curves meet.

(b) Find the area between these two curves.

Exercise 4

Find the area enclosed by $y = 1$, $y = 1 - x^3$, and $y = x^2/2 - 1/2$:



(i) Solve the two equations give x as a function of y--$x = f(y)$

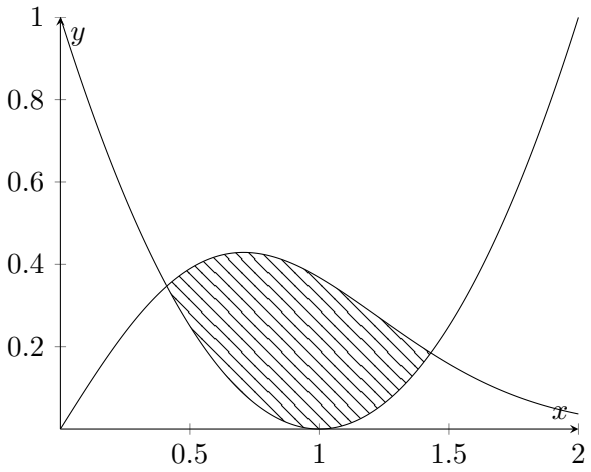
(ii) Set this up an integral of the form $\int_{x_0}^{x_1} f(y) - g(y)dy$.

(iii) Solve the integral to find the area.

Exercise 5

We've just seen that some areas can be found by integrating "horizontally", using y as the independent variable.

This is the region enclosed by $y = xe^{-x^2}$ and $y = (x-1)^2$:



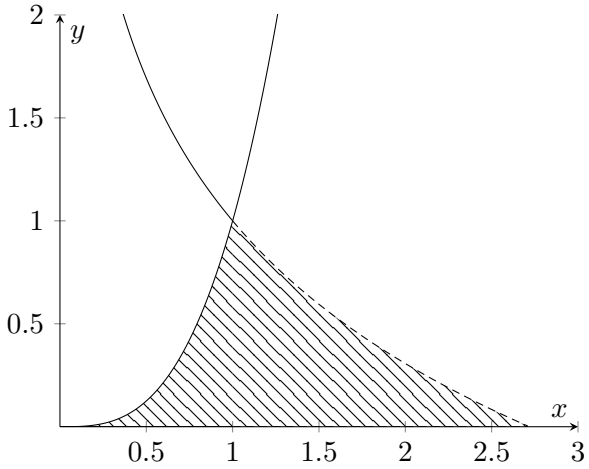
(a) Looking at this region, will it be easier to find by integrating over x or over y? Why?

(b) Set up (but don't solve!) the integral to calculate this by integrating over dx (you can guess at the numeric values to set up the bounds, since you don't need exact values to set up the integrals).

(c) Set up (but don't solve!) the integral to calculate this by integrating over dy (you can guess at the numeric values to set up the bounds, since you don't need exact values to set up the integrals).

(d) Were you right about which version is easier?

This is a region in the first quadrant enclosed by $y = x^3$ and $y = 1 - \ln x$:



(e) Looking at this region, will it be easier to find by integrating over x or over y? Why?

(f) Set up (but don't solve!) the integral to calculate this by integrating over dx (you can guess at the numeric values to set up the bounds, since you don't need exact values to set up the integrals).

(g) Set up (but don't solve!) the integral to calculate this by integrating over dy (you can guess at the numeric values to set up the bounds, since you don't need exact values to set up the integrals).

(h) Were you right about which version is easier?

Exercise 6

A proof reader can read $15xe^{-0.25x^2}$ pages per hour, where x is the number of hours already worked. If a proof reader works for four hours straight one morning (starting from being well-rested), how many pages will the proof reader read?

- In exercise 1, the area is about 1.4667
- In exercise 2, the area is about 0.8333
- In exercise 3, the area is about 31.821
- In exercise 4, the area is about 0.648
- In exercise 6, the proof reader can read about 29.45 pages