Math 103, Fall 2014
Week 1

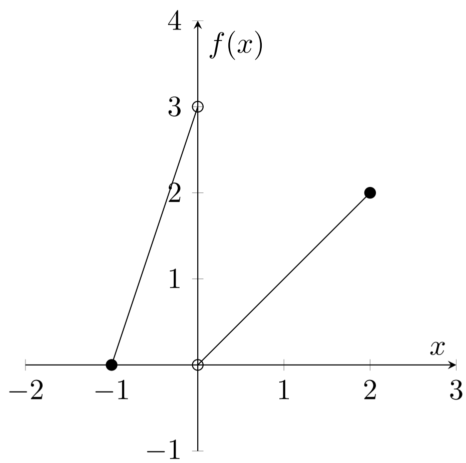
In Class Work, Thursday, August 28th

Welcome to Math 103!
Every day in class, you're going to arrive and get a worksheet like this.

Warm Up

Most days will start with one or two warm up problems. You should get out a piece of paper and start working on these.

Consider the function $f(x)$ shown in the following graph:



(i) Is $f(0)$ defined?

Warm up problems will either be similar to the work you did before class, or, like today, things you're supposed to have seen in previous classes. That doesn't always mean they'll be easy: if you're having trouble you should talk to the other people in your group or put up a red flag to signal that you need help from the professor or TAs.

(ii) What is the domain of $f(x)$?

(iii) What is the range of $f(x)$?

(iv) This function is made up of two lines. What is an equation for the line on the left?

`Sometimes worksheets will have hints written in this font. They might remind you of things you may have forgotten, or break a problem into steps for you, or (like this problem) both. (You're free to ignore the hint if you don't need it or prefer a different way.)`

(a) There are a number of ways to give the equation for a line; for now we'll use the most popular, "slope-intercept form", which is an equation of the form $y = mx + b$.

(b) In order to find the equation for a line, you usually need two points on the line. What are two points on this line?

(c) The value m is the *slope*, and it's equal to

$$\frac{\texttt{the difference between the two y values}}{\texttt{the difference between the two x values}}.$$

(d) The value b is the *intercept*, and it's equal to the value where the line crosses the y-axis.

(e) Put this information together to give an equation for the line.

(v) What is an equation for the line on the right (a function in the form $y = mx + b$ describing this line)?

Hints are like training wheels: they help you learn how to do a problem, but the goal is to solve similar problems without them. For instance, the first time you see a problem, I might break it up into steps for you. But then you might get a similar problem where you're supposed to break it into steps on your own.

Transition to Working as a Group

- Take a minute or two to meet the other people in your group. If you haven't already, get a nametag, fill it out, and it put it on. In theory you'll be working with these people for about two weeks. (In practice, the tends to be some people joining or leaving the class, so we'll have to adjust the groups a bit until enrollment stabilizes.)

- You probably won't all finish the warm ups at exactly the same time. If one person is behind—maybe they came in late, or are just having trouble with this particular warm up—it's okay if not everyone finishes the warm ups, but at least two of you should, and if the third person is close, give them another minute or two.

- Every day you should compare solutions to the warm up and make sure you all agree. If someone seems really confident, or you're feeling unsure, don't just give up on your answer. You should all come to an agreement not only on the final answer, but on what appropriate methods for solving these problems are.

- The main activity is working on problems as a group. You will write up a single solution to the rest of the problems collectively. The solutions will be collected and photocopied so that you can all get a copy. (I'll take a look at them, but they're not graded.)

- Keep the act of solving problems separate from writing down a solution. For now you should all have scratch paper out to do work on.

Exercise 1

We're not quite done with the function $f(x)$ on page 1. Write a formula for $f(x)$ as a piecewise-defined function. (You may want to look at page 5 in the book for what it looks like to write down a formula for a piecewise-defined function.)

`There should be two parts to your piecewise function, each with a corresponding domain.`

Work on this problem as a group and come to an agreement about what the right answer is. You should make sure that every single person in the group understands the solution. Once you've agreed, decide who's going to write up your official solution, and write it up.

When you're done with this problem, put up a green flag on the pole in the middle of your table.

Exercise 2

Consider the function

$$g(x) = \begin{cases} -x^2 + 1 & \text{if } -2 < x \leq 1 \\ x - 4 & \text{if } 1 < x < 3 \end{cases}$$

(a) Is $g(0)$ defined?

(b) What is the domain of $g(x)$?

(c) What is the range of $g(x)$?

(d) Sketch the graph of $g(x)$.

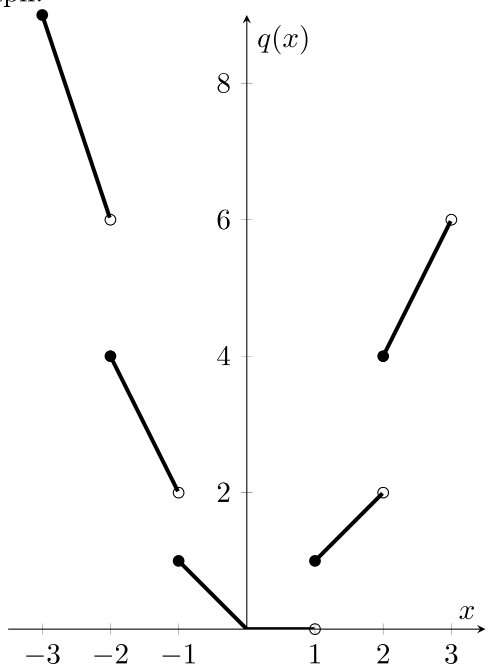
`Make sure to clearly indicate which endpoints` g `is defined at and which endpoints it is not defined at.`

Again, work on this problem as a group, discussing each part in turn. If you get stuck, or you're confused, or you just have a question, raise your hand or use the flags on the table. We're here to help you!

In general, your group should vary who writes up solutions to problems, and everyone should write some up. You could decide to rotate each problem, or have one person be the recorder each day, or some other scheme, as long as everyone participates.

Exercise 3

The function $q(x)$ is defined everywhere. Part of it is shown in the following graph:



(a) Write a formula as a big piecewise defined function with lots of pieces. (It would need infinitely many parts, so exercise judgement about how many parts are enough to convey the pattern.)

(b) Check out the greatest integer and least integer functions on page 5. Using one of those functions, write a single expression for $q(x)$.

This is a bit trickier. Now is a good time to remind you that you can ask for help if you get stuck. Also, it's okay if you don't finish the whole sheet—most of them are intended to be a bit longer than there's time for, in case a group ends up going really fast. If you're really going too slowly, we'll check in on you. It's not *okay to skip problems; they're in the order they are for a reason.*

Remember that everyone in your group should be involved and should be comfortable with the answer and how you got it. If, hypothetically, there were to be a class discussion, and someone in your group were to be randomly selected to explain your group's answer, they should be able to.

(c) Check your formula: pick a couple of values of x where you know the value of $q(x)$ and plug them into your formula to make sure it's right.

The solutions you write down should be solutions, not just the final answer. They should include intermediate steps and the steps you take to verify your answer. They should not *include false starts or mistakes you made along the way. (You should expect to have false starts and to make mistakes—that's fine!—you just shouldn't include them when you write a solution.) That means you shouldn't begin writing a solution until you've finished a problem; you should do your work on scratch paper and write the solution after.*

When you're done with this problem, put up a green flag on the pole in the middle of your table.

Exercise 4

(a) Draw a *new* **odd** function you have never seen before. Make sure to include $[-2, 2]$ in the domain. We'll call this function $h(x)$.

(You don't need to be able to write down an expression for it—in fact, it's better if you pick one which is just a picture, where you don't know an expression.)

(b) Sketch the graph of $3h(x)$.

(i) Find $h(-2)$, $h(-1)$, $h(0)$, $h(1)$, and $h(2)$.

(ii) Now find the values $3h(-2)$, $3h(-1)$, $3h(0)$, $3h(1)$, and $3h(2)$.

(iii) Plot those points on a new graph.

(iv) That may be enough to fill in the graph. If not, fill in a more points--$3h(-1.5)$, $3h(.5)$, and so on--until you think you know what the sketch should look like.

(c) Sketch the graph of $h(x + 2)$.

Notice that I'm not breaking this into steps again, because now you can break it into steps for yourself.

Do check your answer: for instance, based on your original graph, what is $h(-1.5+2) = h(.5)$? Is that the value you sketched at $x = -1.5$ on the new graph?

(d) Sketch the graph of $h(x) - 1$.

(e) Sketch the graph of $h(2x)$.

Exercise 5

Suppose $u(x)$ is an even function and $v(x)$ is an odd function, and this is all you know about these two functions.

(a) Is this enough information to conclude that $u + v$ is odd?

`You should actually give some kind of argument (not necessarily a long or complicated one) here that other students in the class would understand. It should be convincing: suppose that some other group came to a different answer; your solution should be persuasive enough that this other group would change their mind. One of you (but only one--talk about who it should be) should take the role of` *skeptic* `and think about what that other group might say in response to your argument.`

(b) Is this enough information to conclude that $u \cdot v$ is odd?