Math 103, Fall 2014
Week 2

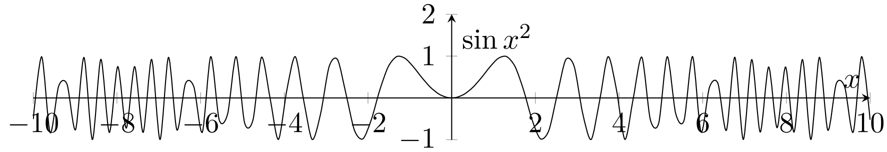
In Class Work, Tuesday, September 2nd

Warm Up

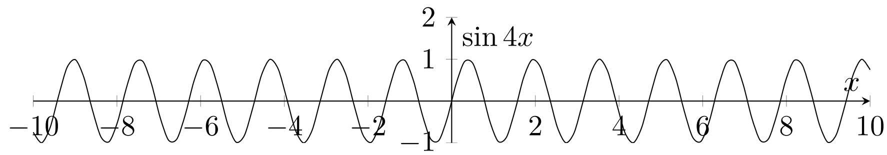
Write your warm up answers wherever you prefer—right on here, or on a separate piece of paper. You can talk to your group while working on them, or wait until your done and compare answers.

(a) Which of these functions are periodic?

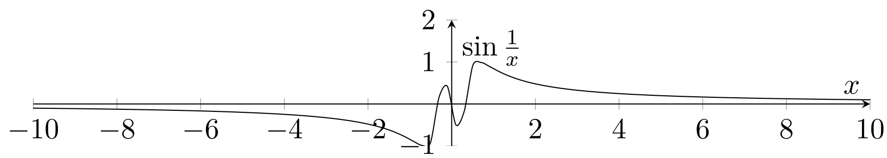
(i)



(ii)



(iii)



(b) Simplify each of the following expressions using the Rules for Exponents:

(i) $\left(\sqrt{1+3x}\right)^3$

(ii) $\left(\sqrt{1+3x}\right)^4$

(iii) $\frac{1+3x}{(\sqrt{1+3x})^3}$

Remember to compare answers with your group.

For the exercises, your group should produce a single collective answer. You should all have your own papers out for trying things out, and once you've all agreed that a solution works, one of you should write it down as your official answer.

Today we will use trigonometric and exponential functions to model physical situations. On each problem we'll have to decide whether trig functions (specifically sin or cos) or exponential functions are an appropriate choice and then modify the functions to match the situation.

Make sure to take a minute to read the question carefully and make sure you understand what it means and what it's asking for.

Exercise 1

Tides are often measured relative to the *MLLW* (Mean Lower Low Water): essentially, the average level of the low tide. Tides are then measured in feet above MLLW: 0 means the MLLW, 7 means 7 feet above the MLLW, and so on.

Our goal is to find a function which approximately describes the water level of the Schuykill at the Market Street bridge. We'll need to work towards this in steps.

(a) Let us first suppose that the Schuylkill's high tides are at midnight and noon (so the low tides are at 6:00 am and 6:00 pm). We'll assume that the two low tides have the same water level, namely 0 feet above MLLW (by definition) and that both high tides have the same water level, 2 feet above MLLW. Write down a function $h_1(t)$ representing the height of the Schuylkill at time t under these assumptions.

(i) Before starting, write down three times at which you know the correct value of $h_1(t)$. These can be used to test whether a formula is correct.

(ii) Sketch a simple graph of this situation. It doesn't have to be very good, but it should capture the most important features of the problem: the way the tides repeat every day and the time and size of the high and low tides.

(iii) Decide on a base function to use. Today we know to choose either $\sin$, $\cos$, or an exponential.

(iv) Write down a guess for the function $h_1(t)$.

(v) `Check your guess: plug in the known values you picked above and see if your function gives the right values.`

(vi) `If your function isn't right, use a graphing calculator to graph your guess. How does it differ from the correct graph? Use this to modify your guess.`

(vii) Don't start writing a solution until you find the right function. Once you've found it, write down a solution explaining what the correct function $h_1(t)$ is, how you found it, and you checked that it was correct.

(b) Let's continue to assume that the Schuylkill's high tides are at midnight and noon (so the low tides are at 6:00 am and 6:00 pm). Again the two low tides are at 0 feet above MLLW, but now let's say (more realistically) that the two high tides are at 7 feet above MLLW. Write down a function $h_2(t)$ representing the height of the Schuylkill at time t under these assumptions.

(i) Again, before starting write a few values where you know the value of $h_2(t)$ so that you can use them to check your answer.

(ii) Write down a guess for the function $h_2(t)$.

(iii) Check your guess by plugging in the values where you know what $h_2(t)$ should be.

(iv) Once you find a function $h_2(t)$ which checks out, write it down on your solutions.

(c) Another twist: let's now suppose that the high tides are actually at 10:30 am and 10:30 pm, while the low tides are at 4:30 am and 4:30 pm. The low tide is still at 0 feet above MLLW and the high tide is still at 7 feet above MLLW. Write down a function $h_3(t)$ representing the height of the Schuylkill at time t under these assumptions.

Put up a green flag when you're finished with this problem. (But then keep going to later problems; we'll talk about the problem once everyone's finished with it.)

Exercise 2

An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. (So after one hour, 71% is left.)

(a) How much ibuprofen is left after 6 hours? (For this, and all parts of this problem, write a formula and don't compute the numeric value.)

  (i) `How much ibuprofen is left after two hours?`

  (ii) `How much is left after three hours?`

  (iii) `Generalize!`

(b) How much ibuprofen is left after 5.5 hours?

(c) How much ibuprofen is left after π hours?

Exercise 3

A break for something less wordy: Find a value of x so that

$$(\sec x)^2 - 3\sec x = 2.$$

`Try giving a name to` $\sec x$`, say` $z = \sec x$`.`

Exercise 4

In 2010, there were 309.3 million people in the US according to the census, and the population was growing at 0.8% per year. (For simplicity, assume this population is accurate for midnight on January 1, 2010.) How many people were there in the US at midnight today? (Find a formula first, but go ahead and plug the value into your calculator to get an actual value too.)

Exercise 5

A ferris wheel is 50 feet in diameter, with the center 60 feet above the ground. A platypus enters from a platform at the 3 o'clock position. After the platypus gets in, the wheel starts spinning clockwise at a constant speed. It takes 80 seconds for the ferris wheel to make one revolution.

(a) What is the height of the platypus 40 seconds after getting on the ferris wheel?

(i) Draw a picture!

(ii) Yes, the platypus made its way half way up and entered from the side rather than the bottom.

(iii) We don't need a formula to find the answer to this: figure out where the platypus had to be on the picture after 40 seconds.

(b) After 40 seconds, is the platypus moving up or down? We still don't need a formula.

(c) What is the platypus' height 70 seconds after getting on the ferris wheel?

(i) Now we do need a formula. Pick a couple times in addition to 40 seconds where you know what the platypus' height is.

(ii) Sketch a graph based on the times where you know the platypus' height.

(iii) Modify a function you know to get a function $h(t)$ giving the platypus' height after t seconds.

(iv) Check $h(t)$ by plugging in the values from the first part and making sure it gives the right answer. If not, try again.

(v) Figure out how knowing $h(t)$ helps find the answer.

(vi) Your solution should explain you find the function $h(t)$ and check that it's correct as well as giving the final answer.