Math 103, Fall 2014
Week 3

In Class Work, Tuesday, September 9th

Warm Up

(a) $r(x)$ is the function $\frac{x^2-1}{x+1}$.

(i) What is $r(-1)$?

(ii) Sketch a graph of $r(x)$.

(iii) What is $\lim_{x\to -1} r(x)$?

(b) What is $\lim_{x\to 3} \frac{7+\sqrt{x+1}}{4-x}$?

Exercise 1

(a) Suppose f is a function and you know that $\lim_{x\to 2} f(x) = 3$. Is this enough information to figure out what $f(2)$? If not, how could you convince a skeptic that this is not enough information?

`Before starting, pick one person in your group to be the skeptic. Whatever conclusion the other two people come to, the skeptic should try to argue against it.`

(b) Suppose g is a function and you know that $g(6) = 5$. Is this enough information to figure out what $\lim_{x\to 6} g(x)$ is? If so, what is $\lim_{x\to 6} g(x)$? If not, how could you convince a skeptic that this is not enough information?

Put up a green flag when you're done with this question.

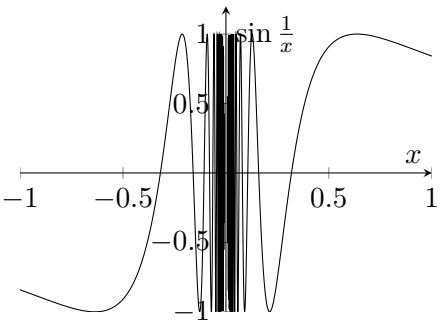
Exercise 2

Consider the limit $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2}$.

(a) Explain (in a sentence or two) why you can't find this just by using the limit laws.

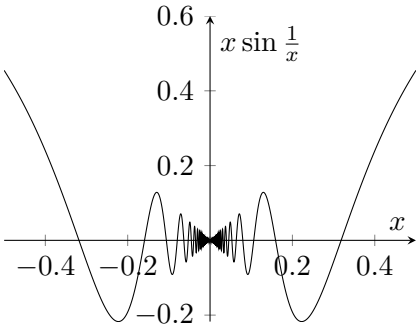
(b) Use algebra to manipulate this into a more convenient form and then find the limit.

Exercise 3



(a) Consider this function: Based on this graph, do you think $\lim_{x\to 0} \sin \frac{1}{x}$ exists or not, and if it does exist, what is its value?

`There's no right or wrong answer at this point. Based on other limits you've seen, what is the most consistent choice, and why? If you've already done the reading for Thursday, you or one of your group members might already know the "official" answer. "The textbook says" is not part of a valid explanation, though you're free to incorporate the reasons the book gives into your argument, in your own words, if you find them persuasive. (You're also free to disagree if you find the book unpersausive.)`



(b) Consider this function:

Based on this graph, do you think $\lim_{x\to 0} x \sin \frac{1}{x}$ exists or not, and if it does exist, what is its value?

`There's no right or wrong answer at this point. Based on other limits you've seen, what is the most consistent choice, and why?`

Put up a green flag when you're done with this question.

Exercise 4

(a) Find the limit $\lim_{h\to 4} \frac{4+h}{5+\sqrt{h^2+9}}$.

(b) Find the limit $\lim_{h\to 4} \frac{4-h}{5-\sqrt{h^2+9}}$.

Exercise 5

Suppose you know that $\lim_{x\to 1} f(x) = 2$.

(a) What is $\lim_{x\to 2} f(x-1)$?

(b) What is $\lim_{x\to 0} \frac{f(x+1)+1}{2}$?

Exercise 6

(a) Suppose you know that $\lim_{x\to 1} \frac{f(x)-2}{x-3} = 2$. What must $\lim_{x\to 1} f(x)$ be?

(b) Suppose you know that $\lim_{x\to 2} \frac{[g(x)]^2+1}{x+1} = 2$. What must $\lim_{x\to 2} g(x)$ be?

(c) Suppose you know that $\lim_{x\to 0} h(x)$ exists and $\lim_{x\to 0} \frac{h(x)}{x} = 1$. What must $\lim_{x\to 0} h(x)$ be?

(d) Suppose you know that $\lim_{x\to 0} r(x)$ exists and $\lim_{x\to 0} \frac{r(x)}{x^2} = 1$. What must $\lim_{x\to 0} \frac{r(x)}{x}$ be?

Exercise 7

We're interested in the functions $u(x) = \frac{1}{x}$ and $v(x) = \frac{-1}{x}$.

(a) Does $\lim_{x\to 0} u(x)$ exist? If so, what is its value?

(b) Does $\lim_{x\to 0} v(x)$ exist? If so, what is its value?

(c) Does $\lim_{x\to 0} u(x) + v(x)$ exist? If so, what is its value?