Math 103, Fall 2014
Week 3

In Class Work, Thursday, September 11th

Warm Up

(a) For each of these functions, identify which points the function is continuous at. (You can use any notation you and the other people in your group find readable.)

(i) $x^3 - 3x^2 + x \sin x$

(ii) $\sqrt{x} + 4x - \cos x$

(iii) $\frac{1}{(x+1)^3} + \frac{2}{(x-2)^2}$

(b) Find $\lim_{x \to 2} \ln \frac{e^x}{4}$ using continuity.

(c) Draw a graph which has a removable discontinuity at $x = 2$, a jump discontinuity at $x = -2$, and which is continuous at all other points.

Exercise 1

Which points is the following function continuous at:

$$h(x) = \begin{cases} x+2 & \text{if } x < -3 \\ x^2 - 2 & \text{if } -3 \leq x < 1 \\ x - 2 & \text{if } 1 \leq x \end{cases}$$

`*After* working this out, graph it (on a calculator if you have one) to check if your answer matches up with the picture.`

Exercise 2

The function

$$g(x) = \begin{cases} x^2 - ax & \text{if } x < 3 \\ ax + 2 & \text{if } x \geq 3 \end{cases}$$

is continuous everywhere. What is a?

`Graph the function you get (on a calculator if you have one) and see if it looks continuous.`

Put up a green flag when you're done with this problem.

Exercise 3

We now have a tool which lets us show that $\lim_{x\to 0^+} x\sin\frac{1}{x}$ does exist. Like other properties of two-sided limits, the Sandwich Theorem works for one-sided limits as well:

Theorem 1 (Sandwich Theorem for Limits from Above). *Suppose that* $g(x) \le f(x) \le h(x)$ *for all* $x > c$. *Suppose also that*

$$\lim_{x\to c^+} g(x) = \lim_{x\to c^+} h(x) = L.$$

Then $\lim_{x\to c^+} f(x) = L$.

(This is almost identical to Theorem 4 on page 72 of the textbook except for the changes to consider the limit from above instead of the two-sided limit.)

Find a number L so that $\lim_{x\to 0^+} x\sin\frac{1}{x} = L$ using the Sandwich Theorem.

(a) Start with the *conclusion* of the theorem--the part after "Then...". How does the conclusion relate to what we're trying to do? In particular, what should c and $f(x)$ be in order to use the Sandwich Theorem?

(b) The Sandwich Theorem calls for two other functions, $g(x)$ and $h(x)$. $h(x)$ is supposed to be a function so that $h(x) \ge f(x)$ whenever $x > c$. Try to find some functions which could be $h(x)$, and which are nicer to work with than $f(x)$. It might help to look at a picture of $x\sin\frac{1}{x}$.

(c) Similarly, try to pick a function $g(x)$ so that $g(x) \le f(x)$ when $x > c$, and so that $g(x)$ is nicer than $f(x)$.

(d) Find the limits of $g(x)$ and $h(x)$. If the limits are equal, explain how the Sandwich Theorem tells you the limit we want. Otherwise, we may have to look for different functions g and h.

Exercise 4

A platypus spend all day on Walnut street, starting at 9am.

- At 9am, the platypus is at the corner of 40th and Walnut.
- At noon, the platypus is in Rittenhouse square, near 19th and Walnut.
- At 1pm, the platypus is in a class in Addams hall around 36th and Walnut.
- Then at 4pm the platypus is at Washington square, around 6th and Walnut.
- Finally, at 11pm, the platypus returns to 40th and Walnut.

(a) Does the platypus ever pass DRL (the math building), at 33rd and Walnut? If so, how many times does the person pass DRL?

(b) Does your answer change if this platypus has a teleportation device?

(c) What does this have to do with continuity?

Exercise 5

Show that the equation $x^4 + 4x^2 - 8$ has a root on the interval $[0, 4]$.

`Remember that a root of the polynomial is a value` c `so that` $c^4 + 4c^2 - 8 = 0$.

Exercise 6

The function u is continuous on $[-2, 2]$, $u(x)$ is never 0, and $u(1) = -1.3$. Is $u(-2)$ positive or negative?

Exercise 7

Show that the equation $x^3 - 5x^2 + 6x - 1$ has at least three roots in the interval $[0, 4]$.