Math 103, Fall 2014
Week 4

In Class Work, Tuesday, September 16th

Warm Up

(a) Find the following limits:

   (a) What is $\lim_{x\to\infty} \frac{3x^2-2x+1}{4x^2+2}$?

   (b) What is $\lim_{x\to-\infty} \frac{x^3-2x^2+7x+1}{4x^3+1}$?

(b) Find the following limits based on their graphs. *Don't* use a graphing calculator. You should know what these graphs look like (or be able to figure it out).

   (a) What is $\lim_{x\to\infty} e^{-x}$?

   (b) What is $\lim_{x\to 0^+} \ln x$?

   (c) What is $\lim_{x\to\infty} \ln x$?

Exercise 1

(a) Does $\lim_{x\to\infty} \sin x$ exist? If so, what is it? If not, does it approach $\pm\infty$?

(b) We would like to find $\lim_{x\to\infty} \frac{\sin x}{x}$. Explain why we can't use the limit law for division to find this limit.

(c) Use the Sandwich Theorem to find $\lim_{x\to\infty} \frac{\sin x}{x}$.

(i) Identify functions $g(x)$ and $h(x)$ so that $g(x) \leq \frac{\sin x}{x} \leq h(x)$ when x is large enough.

(ii) What does the Sandwich Theorem say in this situation?

(d) Graph $\frac{\sin x}{x}$ on a graphing calculator to make sure the result you got is reasonable.

Exercise 2

(a) Graph $\frac{2x}{4x+8}$ (for instance, on a graphing calculator).

(b) Based on the graph, what is $\lim_{x\to -2^+} \frac{2x}{4x+8}$.

(c) We want to find $\lim_{x\to 3^-} \frac{x}{x^2-9}$ without using a graphing calculator.

(i) What is $\lim_{x\to 3^-} x$?

(ii) What is $\lim_{x\to 3^-} x^2 - 9$?

(iii) As $x \to 3^-$, is the sign of $\frac{x}{x^2-9}$ positive or negative?

(iv) What is $\lim_{x\to 3^-} \frac{x}{x^2-9}$?

(v) Use a graphing calculator to check your answer.

(d) What is $\lim_{x\to 3^+} \frac{x}{(x-3)^2}$?

(e) What is $\lim_{x\to 3^-} \frac{x}{(x-3)^2}$?

(f) What is $\lim_{x\to 3} \frac{x}{(x-3)^2}$?

(g) What is $\lim_{x\to 0} \frac{1}{\sqrt{x^2+1}-1}$?

Exercise 3

(a) What is $\lim_{x\to 0^+} \frac{x}{\sqrt{x^2+1}-1}$?

(i) `The first thing we should try is looking at the limits of the top and the bottom and seeing if either the Limit Law for division or the Fractions Approaching Infinity rule apply. Explain why neither does.`

(ii) `Use algebra to convert this limit into another form which you can solve.`

(b) What is $\lim_{x\to 0^-} \frac{x}{\sqrt{x^2+1}-1}$?

(c) What is $\lim_{x\to \infty} \frac{x}{\sqrt{x^2+1}-1}$?

(d) What is $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2+1}-1}\right)$?

Exercise 4 (on Section 2.4)

(a) What is $\lim_{x\to 2^-} \frac{x-2}{|x-2|}$?

(b) What is $\lim_{x\to 2^+} \frac{x-2}{|x-2|}$?

(c) What is $\lim_{x\to 2} \frac{x-2}{|x-2|}$?

Exercise 5 (on Section 2.4)

Recall the limits $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$. Use these, together with the limit laws, to find the following limits.

(a) $\lim_{x\to 0} \frac{\tan x}{x}$

(b) $\lim_{x\to 0} \frac{\sin^2 x}{x^2}$

(c) $\lim_{x\to 0^+} \frac{\sin x}{\sqrt{x}}$