Math 103, Fall 2014
Week 4

In Class Work, Thursday, September 18th

Exercise 1 (from Section 2.4)

If your group finished this problem on Tuesday, skip to the next one.

(a) What is $\lim_{x \to 2^-} \frac{x-2}{|x-2|}$?

(b) What is $\lim_{x \to 2^+} \frac{x-2}{|x-2|}$?

(c) What is $\lim_{x \to 2} \frac{x-2}{|x-2|}$?

Exercise 2 (from Section 2.4)

Recall the limits $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$. Use these, together with the limit laws, to find the following limits.

(a) $\lim_{x \to 0} \frac{\tan x}{x}$

(b) $\lim_{x \to 0} \frac{\sin^2 x}{x^2}$

(c) $\lim_{x \to 0^+} \frac{\sin x}{\sqrt{x}}$

Exercise 3

The limit

$$\lim_{h \to 0} \frac{\sin(\pi + h) - \sin \pi}{h}$$

represents the derivative of the function $\sin x$ at π—that is,

$$\sin'(\pi) = \lim_{h \to 0} \frac{\sin(\pi + h) - \sin \pi}{h}.$$

(Compare to the definition of the derivative to see why.)

Each of the following limits represents the derivative of some function at some point; identify the function and the point. (Don't evaluate the limits.)

(a) $\lim_{h\to 0} \frac{\sqrt{7+h}-\sqrt{7}}{h}$

(b) $\lim_{h\to 0} \frac{(2+h)^2-4}{h}$

(c) $\lim_{h\to 0} \frac{e^r e^h - e^r}{h}$

(d) $\lim_{t\to 0} \frac{\frac{1}{(3+t)^2}-\frac{1}{3^2}}{t}$

Exercise 4

I don't usually like to tell you what method you have to use, but I'm making an exception today, because this problem is really practice on limits for the upcoming exam.

Use the definition of the derivative to find each of the following derivatives:

(a) $f'(3)$ where $f(x) = \frac{1}{x^2}$
`First express this derivative as a limit, then use what you already know about limits to evaluate it.`

(b) $\sin'(\pi/2)$

(c) $g'(4)$ where $g(x) = \sqrt{x}$

(d) $h'(0)$ where

$$h(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Exercise 5

A car is traveling on a widing road at varying speed. Let $g(t)$ represent the distance, in miles, that the object has traveled after t seconds. Let $h(t)$ represent the distance in miles the same object has traveled after t hours.

(a) Write an equation relating the functions g and h.

(i) `This will be much easier if we think about it less abstractly. Make up an example function` g `(make it simple, but not too simple)`

(ii) Use the function g you chose to calculate how far the object travels in one hour

(iii) Now use the function g to calculate how far the object travels in two hours

(iv) And one and a half hours...

(v) Try to generalize to find the relationship between g and h

(vi) Now try to come up with the general relationship between g and h which works no matter what g is.

(b) Write an equation relating g' and h'.

Pick an example which could be g' to think about. (It doesn't have to be the derivative of g you used above--this is a *different* example.) Again, make sure g' is simple but not too simple. It may help to identify what the units of g' and h' are.

(c) This equation should allow you to compute $h'(1)$ if you know some values of $g'(x)$. Which values of x do you need to know $g'(x)$ for in order to compute $h'(1)$?

Exercise 6

Sketch a function $y(x)$ satisfying these conditions:

- $y(1) = 2$
- $y'(1) = -2$
- $y(3) = 3$

Exercise 8

Let $f(x) = |x|$.

(a) Does f have a derivative at $x = 2$? If so, what is it? If not, why not?

Give an explanation in a few sentences which would convince a classmate. As always on these problems, it's a good idea to have someone in your group play skeptic.

(b) Does f have a derivative at $x = 0$? If so, what is it? If not, why not?