Math 103, Fall 2014
Week 5

In Class Work, Tuesday, September 23rd

Exercise 1

A few groups started on this problem last week, but none of them got it right. I've tweaked the instructions a bit, so even if you tried this before, start it over with a fresh eye.

A car is traveling on a widing road at varying speed. Let $g(t)$ represent the distance, in miles, that the object has traveled after t seconds. Let $h(t)$ represent the distance in miles the same object has traveled after t hours.

(a) Write an equation relating the functions g and h.

`This is a question we have plenty of intuition for, but it's hard to bring that intuition to bear on an abstract function like` $g(t)$`. A useful technique is to first think about a concrete example. We'll make up an example function which could be` $g(t)$ `and think about that example before trying to think about the general case. We have to pick a concrete function to stand in for` $g(t)$`; if it's too complicated, it won't be any better than an unknown function, but if it's too simple, it won't be representative. Picking good examples is an art; in this case, I happen to know that a linear function will be too simple, but that anything more complicated will work.`

(i) `Make up an example function` g`. It shouldn't be linear, but shouldn't be too much more complicated. Some good options are a quadratic (if you want to think about the car as travelling in one direction and accelerating) or sin/cos (if you want to think about the car going back and forth).`

(ii) `Use the function` g `you chose to calculate how far the object travels in one hour`

(iii) `Now use the function` g `to calculate how far the object travels in two hours`

(iv) `And one and a half hours...`

(v) `Try to generalize to find the relationship between` g `and` h

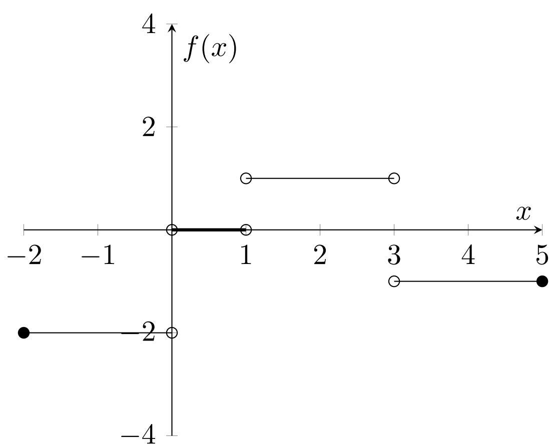
(vi) `Now try to come up with the general relationship between` g `and` h `which works no matter what` g `is.`

(b) Write an equation relating g' and h'.

`Again, a good way to start is by picking a concrete example which could be` g'`. It doesn't have to be the derivative of` g `you used above--this is a` ***`different`*** `example. This time it turns out that even a linear function is complicated enough. When figuring out the relationship between` g' `and` h'`, it may help to identify what the units of` g' `and` h' `are.`

(c) This equation should allow you to compute $h'(1)$ if you know some values of $g'(x)$. Which values of x do you need to know $g'(x)$ for in order to compute $h'(1)$?

Exercise 2



(a) Plot f assuming that $f(-2) = 1$.

(b) Plot f on the same axes, this time assuming $f(-2) = 4$

Exercise 2

We will study functions

$$g(x) = \begin{cases} ax + b & \text{if } x \leq 1/2 \\ x^2 & \text{if } x > 1/2 \end{cases}$$

where a and b are constants.

(a) What relationship must hold between a and b in order for $g(x)$ to be continuous?

(b) What values must a and b have in order for $g(x)$ to be differentiable as well?

(c) Have each person in your group draw a different version of $g(x)$ (that is, a different choice of a, b) so tha $g(x)$ is continuous but not differentiable.

(d) Draw the version where $g(x)$ is differentiable.

(e) Compare the graph where $g(x)$ is differentiable to the three where it isn't. Write a sentence or two describing the difference.

Exercise 3

The height of a certain projectile in feet above ground level is given by $h(t) = 192t - 16t^2$ where t is measured in seconds after firing. At what time is the projectile neither ascending nor descending? (You don't need to calculate the derivative! Use what you know about quadratic functions.)

Exercise 4

The limit

$$\lim_{z\to\pi} \frac{\sin(z) - \sin\pi}{z - \pi}$$

represents the derivative of the function $\sin x$ at π—that is,

$$\sin'(\pi) = \lim_{z\to\pi} \frac{\sin(z) - \sin\pi}{z - \pi}.$$

(Compare to the alternative formula for the derivative to see why.)

Each of the following limits represents the derivative of some function at some point; identify the function and the point. (Don't evaluate the limits.)

(a) $\lim_{z\to 7} \frac{\sqrt{z}-\sqrt{7}}{z-7}$

(b) $\lim_{z\to 2} \frac{z^2-4}{z-2}$

(c) $\lim_{z\to r} \frac{e^z-e^r}{z-r}$

(d) $\lim_{h\to 3} \frac{\frac{1}{h^2}-\frac{1}{9}}{h-3}$

Exercise 5

Use the alternative form of the derivative to find each of the following:

(a) $f'(2)$ where $f(x) = x^2$,

(b) $g'(7)$ where $g(x) = \sqrt{x}$,

(c) $u'(0)$ where $u(x) = \cos x$.