Math 103, Fall 2014
Week 5

In Class Work, Thursday, September 25th

Exercise 1

Section 3.3 introduces several rules for calculating derivatives which are easier to use than constantly going back to the definition in terms of a limit. These rules can be found in blue boxes throughout section 3.3.

Use rules on pages 137 and 138 (the Power Rule and Constant Multiple Rule) to find each of the following derivatives.

(a) $\frac{d}{dx}x^7$

(b) $\frac{d}{dx}x^\pi$

(c) $\frac{d}{dt}t^{-2}$

(d) $\frac{d}{dz}\sqrt{z^3}$

(e) $\frac{d}{dx}7x^2$

(f) $\frac{d}{dt}\frac{-1}{2}x^e$

(g) $\frac{d}{dx}3u(x)$ where $u(x)$ is an unknown function with $u'(x) = e^{-x^2}$.

Exercise 2

The Sum Rule and Product Rule (pages 138 and 141, respectively) tell us how to combine derivatives we know to find more derivatives. Use them, together with the Power Rule, Constant Multiple Rule, the Derivative of a Constant (page 136), and the Derivative of the Natural Exponential (page 140) to find the following derivatives:

(a) $\frac{d}{dx}(3x^2 + 7x)$

(b) $\frac{d}{dt}(\frac{1}{t^2}+3)$

(c) $\frac{d}{dx}x^2e^x$

Exercise 3

Find the derivative $\frac{d}{dz}[(z+1)(z+2)]$ two different ways.

(a) By using the product rule.

(b) By expanding the product and then using the sum rule.

Exercise 4

Since the derivative of a function is another function, we can find higher-order derivatives by taking the derivative multiple times. (See page 143 in the textbook.) Note that, just like with regular derivatives, we have two different ways of writing multiple derivatives.

Find the following higher-order derivatives:

(a) $\frac{d^3}{dx^3}x^7$

(b) $f^{(4)}(x)$ where $f(x) = e^x$

Exercise 5

Let f be any differentiable function.

(a) Write an expression for $\frac{d}{dx}(e^x\, f(x))$.

(b) Write an expression for $\frac{d^2}{dx^2}(e^x\, f(x))$.

(c) Check your formula by finding $\frac{d^2}{dx^2}(xe^x)$ in two different ways—once by plugging into your expression from the previous part, and once directly using the product rule.

Exercise 6

Suppose that $u(x)$, $v(x)$, and $g(x)$ are all functions and that $g(x)\cdot v(x) = u(x)$.

(a) Differentiate this identity (that is, differentiate both sides with respect to x).

(b) Solve this equation for g'.

(c) Simplify, if necessary, into a single fraction. Compare the result to the Quotient Rule (page 142). What is the relationship, and why?

Exercise 7

Suppose $g(2) = 5$, $g'(2) = 4$, $h(2) = 3$, and $h'(2) = 7$.

(a) What is $\frac{d}{dx}(g(x)h(x))\Big|_{x=2}$?

(b) What is $\frac{d}{dx}(g(x)h^2(x))\Big|_{x=2}$?

(c) What is $\frac{d}{dx}\frac{g(x)}{h(x)}\Big|_{x=2}$?

Exercise 8

A company provides tech support to purchasers of its product. If the company has sold p products, it will receive $c(p)$ calls per month, and each of those calls will take an average of $t(p)$ minutes.

(a) Write an equation for $M(p)$, the average total number of minutes of tech support the company needs to provide each month, in terms of $c(p)$ and $t(p)$.

(b) If the number of customers increases starts at p and increases by Δp, the number of calls will change by Δc and the average length by Δt. Write formulas for Δc and Δt in terms of $c(p)$, $c(p + \Delta p)$, $t(p)$, and $t(p + \Delta p)$.

(c) Write a formula for ΔM, the change in the minutes per month, in terms of $c(p)$, Δc, $t(p)$, and Δt.

(d) Use the product rule on your formula for $M'(p)$ to find $M'(p)$ in terms of $c(p)$, $c'(p)$, $t(p)$, and $t'(p)$.

(e) Compare your formula for $M'(p)$ to your formula for ΔM.