Math 103, Fall 2014
Week 6

In Class Work, Tuesday, September 30rd

Warm Up

(a) Let $s(t)$ be the speed of a train in MPH at time t seconds after it leaves 30^{th} Street Station. What is the interpretation of $\frac{ds}{dt}$ and what are its units?

(b) Water is leaking out of a tank at a variable rate, leaking at rate $f(t)$ at time t.

  (a) What are reasonable units for $f(t)$ and t?

  (b) Suppose that $g'(t) = f(t)$. What is an interpretation for $g(t)$ and what are its units?

(c) Find $\frac{d}{dz}\frac{\tan z + e^z}{\sin z}$.

Exercise 1

In a simple economic supply and demand model for the price of gasoline, there is a **demand function** g such that setting the price of gas at x per gallon results in total consumption of gas equal to $g(x)$ gallons. What is an interpretation of the quantity $\frac{d}{dx}(x \cdot g(x))$? (This doesn't have a pithy name; just describe it in words.)

Exercise 2

At constant temperature, the ideal gas law says that $PV = c$ where c is a constant, P is the pressure of the gas, and V is the volume taken up by the gas. (Pressure is measured in $kg/m \cdot s^2$, volume in m^3, and the constant has units $kg \cdot m^2/s^2$.)

(a) Find $\frac{dP}{dV}$ (including units)

(b) Find $\frac{dV}{dP}$ (including units)

Exercise 3

Write an equation representing the following Malthusian population law.

> "The rate of population growth is always proportional to the amount of population."

(i) `Start by picking a variable to represent the population at time` t `(that is, as a function of` t`)`

(ii) `Figure out how to express "the rate of population growth"`

(iii) `Figure out how to express "the amount of population"`

(iv) `Write down the equation connecting the two. (You may have forgotten the definition of "proportional". It's way back on page 7!)`

Exercise 4

> "The downward acceleration of a falling object is equal to the gravitational constant minus an air resitance proportional to the speed of the object."

(a) Write an equation for this modified gravitational law, giving units and an interpretation of all functions, variables and constants.

(b) In the notation just established, write an equation stating that "The falling object is not accelerating."

(c) The speed of the object when it is no longer accelerating is the **terminal velocity**. What is the terminal velocity in terms of the constants you have defined?

Exercise 5

What is $\frac{d^{191}}{dx^{191}} \sin x$?

Exercise 6

The limit

$$\lim_{z \to \pi/6} \frac{\sec(z) - \sec(\pi/6)}{z - \pi/6}$$

is equal to the derivative of some function at some point. Use this definition and the formula for the corresponding derivative to calculate this limit.

Exercise 7

(a) Write down a trigonometric identity relating $\sin^2 x$ to $\cos^2 x$.

(b) Take the derivatives of both sides of this identity to obtain an identity involving $\frac{d}{dx} \sin^2 x$ and $\frac{d}{dx} \cos^2 x$. What do you learn about the relationship between $\frac{d}{dx} \sin^2 x$ and $\frac{d}{dx} \cos^2 x$?

`Leave` $\frac{d}{dx} \sin^2 x$ `and` $\frac{d}{dx} \cos^2 x$ `alone--we want an equation that includes those two derivatives.`

(c) Calculate the derivatives of $\sin^2 x$ and $\cos^2 x$ and verify that the identity you discovered above holds.