Math 103, Fall 2014
Week 6

In Class Work, Thursday, October 2nd

Warm Up

(a) The chain rule is used to find the derivative of functions which are computed in multiple steps. For example, the function $e^{\sin x}$ is computed by first finding the intermediate value $u = \sin x$, and then calculating e^u. Break each of the following functions into a composition of two separate simpler functions. (There's no need to take the derivative.)

  (i) $e^{\tan x}$

  (ii) $\sin e^x$

  (iii) $\tan^{-1}(1 + x^3)$

  (iv) $\ln \cos^{-1} \sqrt{x}$

  (v) $\ln \tan e^{x^2}$

(b) (i) Find $\frac{d}{dx} e^{\tan x}$.

  (ii) Find $\frac{d}{dx} \sin e^x$.

Exercise 1

(a) What is $\frac{d}{dx} \sin e^{x^2}$?
`Anything written in this font is only there to provide hints to get through a problem. You're always free to ignore them if you don't find them necessary. i, ii, and iii aren't separate questions, they're just steps you might want to take while solving a.`

   (i) `First write this as a composition of two functions:` $\sin e^{x^2} = f(g(x))$

   (ii) `Find` $f'(u)$ `and` $g'(x)$

   (iii) `Use the chain rule to find the answer.`

(b) What is the derivative with respect to x of $e^{\sqrt{1+x^2}}$?

Exercise 2

(a) What is $\frac{d}{dx} \ln \sin x$?

(b) What is $\frac{d}{dx} \sqrt{\ln \sin x}$?

(c) What is $\frac{d}{dx} \tan \sqrt{\ln \sin x}$?

(d) What is $\frac{d}{dx} e^{\tan \sqrt{\ln \sin x}}$?

Exercise 3

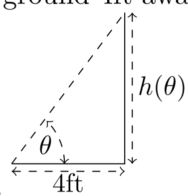
A car is traveling on a winding road at a varying speed. Let $g(s)$ represent the distance, in miles, that the object has traveled after s seconds. Let $h(t)$ represent the distance in miles the same object has traveled after t hours.

(a) Write an equation relating the functions g and h.

(b) Take the derivative to find an equation relating g' and h'.

Exercise 4

A lamp is on the ground 4ft away from a wall and angled upwards at angle θ as in this picture



It casts a beam on the wall at height $h(\theta)$.

(a) What is $h(\theta)$ as a function of θ?

(b) Give the units of $\frac{dh}{d\theta}$ and an interpretation.

(c) What is $\frac{dh}{d\theta}$?

(d) What is $\theta(d)$, the angle in radians as a function of the angle in degrees?

(e) What is $\frac{d}{dd}h(\theta(d))$?

(f) When the angle is at $\pi/4$ radians (equivalently, 45 degrees), is $\frac{d}{dd}h(\theta(d))$ the same number as $\frac{d}{d\theta}h(\theta)$? If not, which is bigger?

Exercise 5

If f, g and h are differentiable functions, compute the following quantities (your answer might include the functions f, g, h and f', g', h', as well as other functions).

(a) $[f(x) \cdot \sin g(x)]'$

`Make sure to test your answer: try an example where you know` f, g, f', g' `and where the derivative of` $f(x) \cdot \sin g(x)$ `is easy to find, and make sure your formula gives the same answer you get by hand.`

(b) $[f(g(h(x)))]'$

(c) $[\ln(f(x) \cdot g(x))]'$

Exercise 6

What is the derivative of 3^x?

Exercise 7

There is a function called erf with the property that $\frac{d}{dx}\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$. What is $\frac{d}{dz}\operatorname{erf}(\sin z)$?

Exercise 8

You know the following values of f, g, f', g':

	1	2	3	4
f(x)	1	2	3	4
g(x)	2	1	3	4
f'(x)	4	3	2	1
g'(x)	3	2	4	1

What is $[f \circ g]'(2)$?