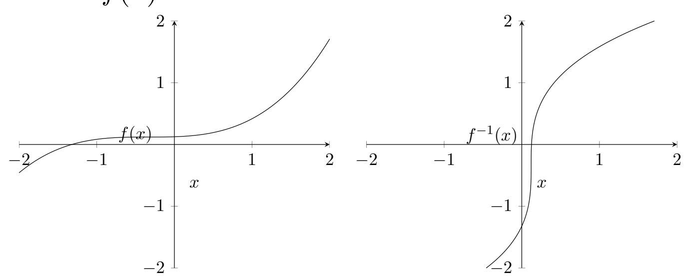
Math 103, Fall 2014
Week 7

In Class Work, Tuesday, October 7th

Warm Up

A function $f(x)$ and its inverse are drawn below.



(i) Between $x = -2$ and $x = 1$, $f(x)$ is nearly flat. What does this mean about its derivative?

(ii) Identify the part of the graph of f^{-1} between $f(-2)$ and $f(1)$. Is the value of $(f^{-1})'$ large or small on this interval? Positive or negative?

(iii) Draw the tangent line to $f(x)$ at $x = -2$. Is the slope positive or negative? Is the absolute value of the slope more than 1 or less than 1?

(iv) Draw the tangent line to $f^{-1}(x)$ at $f(-2)$. Is the slope positive or negative? Is the absolute value of the slope more than 1 or less than 1?

Exercise 1

Let $f(x)$ be the function $f(x) = x^5 + x^2 + 1$. What is $\frac{d}{dx} f^{-1}(x)$? (The answer will include $f^{-1}(x)$.)

(a) Find the answer using implicit differentiation.

- (i) `Set` $y = f^{-1}(x)$ `and solve to get` $x = \cdots$.
- (ii) `Use implicit differentiation on this equation.`
- (iii) `Solve for` $\frac{dy}{dx}$`; you'll end up with` y `as well as` x `in your answer.`
- (iv) `Substitute` $f^{-1}(x)$ `for` y `to get a final answer.`

(b) Find the answer using the formula for derivatives of inverse functions.

Exercise 2

Consider the function $g(x) = \frac{1}{\sqrt{x+1}}$. Below is a calculation of the derivative of $g^{-1}(x)$ using implicit differentiation. Copy the steps into your solution, filling in the missing steps.

- We give the name $y = g^{-1}(x)$, so we are trying to find $\frac{dy}{dx}$.
- Since $y = g^{-1}(x)$, also $g(y) = x$.
- By implicit differentiation, we have $\frac{dy}{dx} =$ _______ (fill in a function of x and y—or maybe just a function of y)
- We know that $x = \frac{1}{\sqrt{y+1}}$, so we can substitute that in to the previous equation to find that $\frac{dy}{dx} =$ _______ (fill in a function of x)

Exercise 3

Suppose you have an unknown function h with the following table of values:

x	1	2	3	4	5	6	7
h(x)	7	5	2	-1	-3	-6	-7
h'(x)	-4	0	-3	-1	-2	-1	-1

(a) What is $h^{-1}(2)$?

(b) What is $h'(h^{-1}(2))$?

(c) Use the derivative rule for inverses (p. 177) to find $(h^{-1})'(2)$.

(d) Use the derivative rule for inverses to find $(h^{-1})'(-3)$.

Exercise 4

Find $\frac{d}{dx}\sqrt{(x^2-4)\sqrt{2x+1}}$.

Taking the derivative directly would take a lot of calculation. Instead, we use *logarithmic differentiation*:

(i) Set $y = \sqrt{(x^2-4)\sqrt{2x+1}}$. Take the logarithm of both sides, and simplify the right (as far as you easily can).

(ii) You now have $\ln y = \cdots$. Take the derivative of both sides using implicit differentiation.

(iii) Solve for $\frac{dy}{dx}$ in terms of x. (Remember you know what y is.) Don't multiply everything out! Leaving it as a product is a perfectly good answer.

Exercise 5

Consider the two functions

- $g(x) = \frac{(x^2+2)e^x}{(x+1)x^3}$, and
- $h(x) = \sin\tan e^x$.

We want to find $g'(x)$ and $h'(x)$.

(a) One of these functions will be easier to find using logarithmic differentiation. Which one, and why?

(b) Find $g'(x)$ and $h'(x)$, using logarithmic differentiation for exactly one of them.

Exercise 6

tanh is a function you may not have encountered yet. $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. Find $\frac{d}{dx} \tanh^{-1} x$.

(i) `Find` $\tanh' x$.

(ii) `Using some algebra, manipulate your definition of` $\tanh' x$ `so that it is expressed in terms of` $\tanh x$.

(iii) `What is` $\tanh' \tanh^{-1} x$? (`tanh`$^{-1}$ `is the inverse of tanh. Remember to use the properties of inverses.)`

(iv) `Use the derivative rule for inverses to find` $\frac{d}{dx} \tanh^{-1} x$?

Exercise 7

In a simple economic model, there is a relationship between the *price* p of widgets and the *quantity* $Q(p)$ of widgets that will be sold at price $Q(p)$.

(a) What are the units of $Q'(p)$? For most goods, is $Q'(p)$ positive or negative?

(b) The *elasticity of demand* is the value $E(p) = \frac{p}{Q(p)} Q'(p)$. This value measures change relative to the original values. For example, if $E(p)$ is always equal to -1, that means that when the price increases by 1%, the quantity will decrease by 1%. What are the units of elasticity?

(c) Suppose $Q(p) = 3p^{-1/2}$. Find $(\ln Q(p))'$.

(d) Remember that $(\ln Q(p))' = \frac{Q'(p)}{Q(p)}$. What is $E(p)$?