Math 103, Fall 2014
Week 8

In Class Work, Tuesday, October 14th

Exercise 1

Someone has drawn the parabola $y = x^2$ in the sand at a beach with the lifeguard's chair at the origin. A person ties one end of a rope to themself and the other end to the chair and begins walking along the parabola so that their x coordinate is changing at a steady rate of 1 meter per minute. They keep the rope pulled tightly so that it forms an angle with the x-axis. How fast is that angle changing when the person has moved 3 meters in the positive x direction?

(i) Draw a picture of this situation.

(ii) This question is telling you a particular rate, $\frac{d?}{dt} = ?$. Which variable is $?$ in this case and what is its value?

(iii) This question is asking you for a particular rate, $\frac{d?}{dt}$. Which variable is $?$ in this case?

(iv) Make sure that both these variables are identified as variables in your picture.

(v) Before doing any math: based on your picture and your intuition, do you expect the final answer to be positive or negative?

(vi) Write down an equation relating these two variables.

(vii) Apply implicit differentiation to this equation to find an equation connecting the two rates we are interested in.

(viii) We are also given a specific fact about a moment of time. What fact is this?

(ix) Combine these last two equations to find the desired rate.

Exercise 2

A 10 ft long ladder leans against a wall so that the angle between the ladder and the ground is 60 degrees. You climb the ladder, but when you reach the top, the ladder starts slipping so that the bottom is sliding away from the wall at a rate of 0.5 ft/sec. How quickly is the angle between the ladder and the ground changing?

(i) For each of the following quantities, identify whether it is constant or a variable:

- The length of the ladder
- The angle between the ladder and the ground
- The distance between the bottom of the ladder and the wall

(ii) Draw a picture

(iii) What rate are we looking for?

(iv) What rate are we given?

(v) Do you expect the answer to be positive or negative?

(vi) Write an equation relating these two variables

(vii) Use implicit differentiation on this equation

(viii) What fact are we given to identify the moment we are interesting?

(ix) Find the rate we want at the correct moment.

Exercise 3

A lighthouse stands 100m away from a straight shoreline with a tall cliff. Its lantern makes a revolution every 15 seconds. When the lantern points in the direction of the shore, the beam creates a spot of light where it hits the cliff. When the beam of light is hitting the shoreline at a right angle, how quickly is this spot of light moving along the cliff?

Make sure to draw a picture and to identify the relevant variables--the rate we want and the rate we're given.

Exercise 4

A spotlight stands on top of a 10 meter pole. A person runs away from the pole at a rate of 20 meters per second. The spotlight turns so that it is always pointing directly at the person's feet. When the person is 50 meters from the spotlight, how quickly is the angle between the spotlight and the pole changing? (Be careful: the person is on the ground and 50 meters from the *spotlight*, which is a small thing on top of the pole.)

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Make sure to break this into steps appropriately!
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Exercise 5

You release a kite in a park. You stand in one place at slowly let out the kite string as the wind blows it up and away. If the wind is blowing it horizontally away from you at a rate of 1 meter per second and blowing it up at a rate of 0.5 meters per second, how quickly are you letting out the kite string?