Math 103, Fall 2014
Week 8

In Class Work, Thursday, October 16th

Warm Up

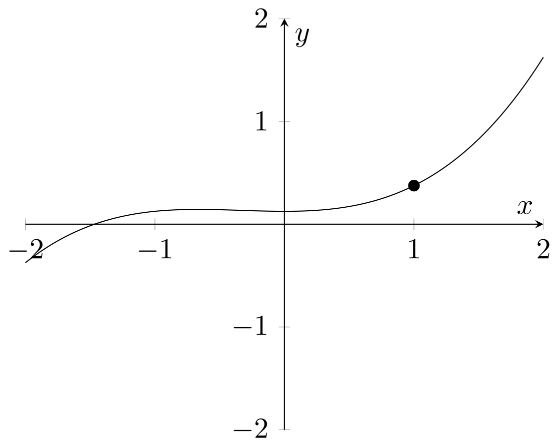
(a) (i) Find the linearization of $f(x) = (8+x)^{1/3}$ at $x = 1$.

(ii) Use a linearization to estimate $\sqrt[3]{8.1}$.

(b) Without using a calculator, estimate $\frac{1}{2.1}$.

Exercise 1

Consider the function $h(x)$ in the graph below:



(a) Draw the linearization at the marked point.

(b) Will the linerization give an overestimate or an underestimate for the value of $h(1.1)$?

Exercise 2

We have a hose with water pouring out, and we want to know how narrow the thinnest portion of the hose is. We calculate this by measuring the volume of water V and using the equation

$$r(V) = 2\sqrt[4]{V}.$$

(a) What is $r'(V)$?

The true volume of water is the number V_0. However we make a small mistake in our measurement and instead measure the value $V_0 + \Delta V$.

(b) In this case, what is the estimate of Δr (as a function of V_0 and ΔV)?

(c) Estimate the relative error in this case. (Remember that you know an equation for $r(V_0)$.)

(d) Suppose $\frac{\Delta V}{V} = 0.01$ (a one percent error). What is the relative error in r?

Exercise 3

You are trying to cut an ordinary piece of 8.5”x11” paper to obtain a piece which is 8.5”x6”. You will then fold this into a cylinder which is 8.5” tall with circumference 6”.

(a) Let x be width you actually cut (the value you hope will be 6”), and therefore the circumference of the cylinder. What is the volume of the cylinder as a function of x?

(b) If you cut the paper accurately to within 0.01 inches, what will the relative error in the volume be?

Exercise 4

If you throw a ball at velocity v at an angle θ, the object will land $\frac{v^2 \sin 2\theta}{g}$ meters away (where g is the gravitational constant). We are aiming at a target $\frac{50^2}{g}$ meters away, so if we throw the ball at the angle $\pi/4$ with $v = 50$, we will exactly hit the target. Which throw will get us closer: if we get the velocity exactly right but the angle wrong by 0.2 meters or if we get the angle exactly right but the velocity wrong by 0.2 radians? (Note that you *don’t* need a calculator for this problem.)

Exercise 5

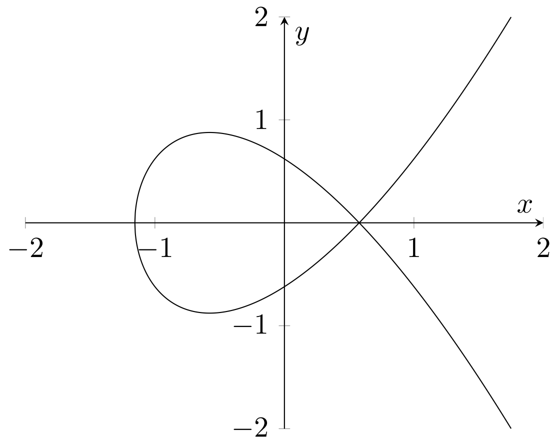
(a) Draw a graph, identify a point, and draw the linearization at that point so that the linearization always underestimates the true value of the function. (That is, $L(x_0 + \Delta x) < f(x_0 + \Delta x)$ if Δx is a small but not 0.)

(b) Draw a graph, identify a point, and draw the linearization at that point so that the linearization always *overestimates* the true value of the function. (That is, $L(x_0 + \Delta x) > f(x_0 + \Delta x)$ if Δx is small but not 0.)

(c) Draw a graph, identify a point, and draw the linearization at that point so that when Δx is small positive number, $L(x_0 + \Delta x) < f(x_0 + \Delta x)$, but when Δx is a small negative number, $L(x_0 + \Delta x) > f(x_0 + \Delta x)$.

Exercise 6

Consider this graph of points satisfying $y^2 = x^3 - x + \sqrt{\frac{4}{27}}$:



(a) Before taking the derivative, use the graph to identify where you think the derivative of this curve will be defined.

(b) Use implicit differentiation to find the derivative $\frac{dy}{dx}$? (Your final answer should be $\frac{dy}{dx} = \cdots$ with only x and y on the right.)

(c) Compare the points where $\frac{dy}{dx}$ isn't defined to the points on the curve where you didn't expect the derivative to be defined.

(d) Use implicit differentiation to find the derivative $\frac{d^2y}{dx^2}$. (Your final answer should be $\frac{d^2y}{dx^2} = \cdots$ with only x and y on the right.)